Quantum Gravity Phenomenology

A systematic approach

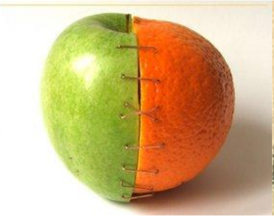
Yuri Bonder

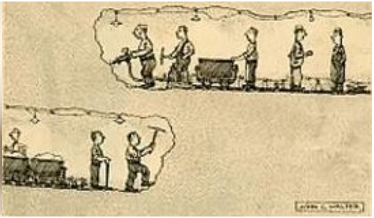
Departamento de Gravitación y Teoría de Campos
Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México

XII Taller de la DGFM-SMF
1 de diciembre de 2017
Guadalajara, Jalisco

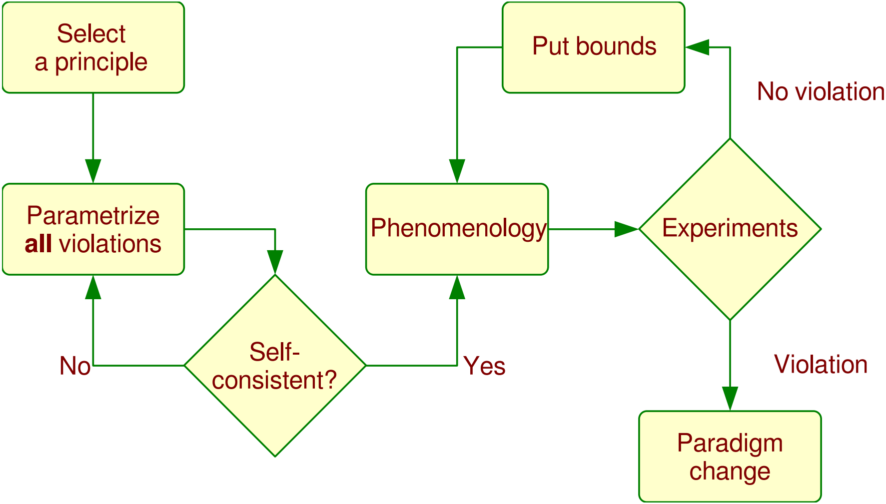
Theoretical physics status

- Fundamental physics = GR + QM.
- Accurate empirical description (where we have access).
- Theoretically inconsistent ⇒ new theory (QG).
- Towards QG: top down *vs.* bottom up.
- No clues on the nature of QG!



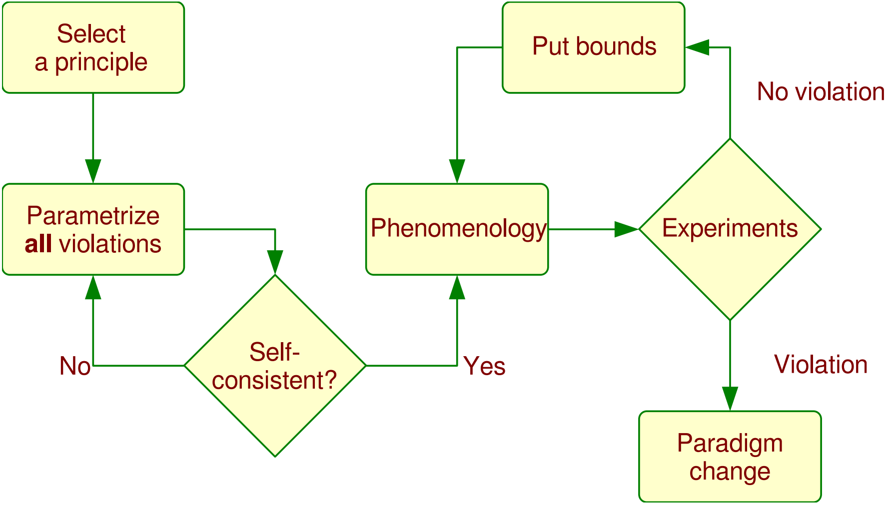


(Idealized) phenomenologists' workflow



- Often, steps 2 and 3 not considered.

Select a principle



GR principles

- Equivalence principle(s).
- Diffeomorphism invariance.
- Local Lorentz invariance.
- Einstein-Hilbert action.
- Torsion-free.
- 4 dims.
- $\vdots$

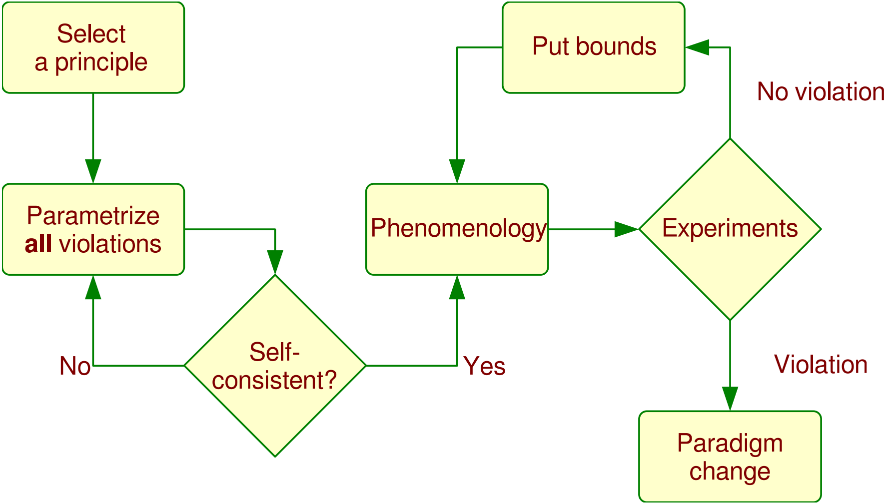
- These principles are not independent.
- In addition, we have the principles of quantum mechanics and the SM.

Lorentz invariance

- As an example, we focus on local Lorentz invariance.
- Lorentz invariance = *all* local inertial frames are equivalent.
- Inertial $\leftrightarrow$ free particles (w.r.t. known interactions).
- No preferred (nondynamical) spacetime directions.
- At the level of the action: inv. under local $SO(1,3)$ "rotations" (tetrads).
- Motivation:
  - LI is fundamental for both GR and QFT.
  - LV includes CPT violation[1].
  - Motivated by spacetime discreteness.
  - Accommodated by most QG candidates (*e.g.*, ST, LQG).
  - Possible discovery of new interactions.
  - Clear phenomenology: perform the same experiment in different frames.

[1] Greenberg PRL 2002

Parametrize **all** violations



Effective field theory

- EFT is useful when the fundamental d.o.f. are unknown.
- Requires knowing the field content and symmetries.
- Field content = standard physics;
  symmetries = standard physics without LI.
- Result: Most general parametrization!
  Lagrange density[1]

$$\mathcal{L} = \mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{LV}}.$$

  where $\mathcal{L}_{\mathrm{LV}}$ contains *all* possible LV additions to SM + GR.
- Naive expectation: $\mathcal{L}_{\mathrm{LV}}$ is suppressed by $E_{\mathrm{EW}}/E_{\mathrm{P}} \sim 10^{-17}$.
- Terms of every dimensionality (higher dimensions more suppressed).

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[1] "Standard Model Extension": Colladay+Kostelecký PRD 1997; PRD 1998; Kostelecký PRD 2004;...

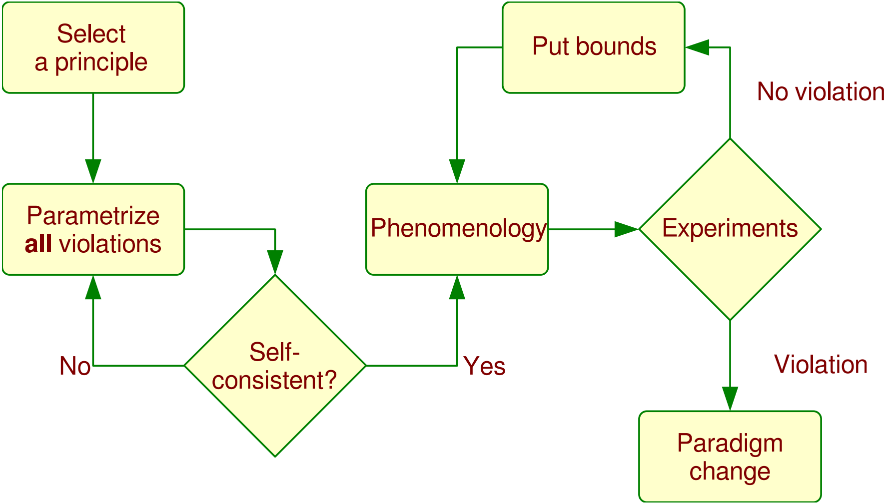
Example: Free Dirac spinor minimal sector in flat spacetime

- Minimal = operators of renormalizable dimension:

$$
\begin{aligned}
\mathcal{L} &= \frac{i}{2}\bar{\psi}\Gamma^\mu\partial_\mu\psi - \frac{i}{2}(\partial_\mu\bar{\psi})\Gamma^\mu\psi - \bar{\psi}M\psi, \\
\Gamma^\mu &= \gamma^\mu - \eta^{\mu\nu}c_{\rho\nu}\gamma^\rho - \eta^{\mu\nu}d_{\rho\nu}\gamma_5\gamma^\rho - \eta^{\mu\nu}e_\nu \\
&\quad -i\eta^{\mu\nu}f_\nu\gamma_5 - \frac{1}{2}\eta^{\mu\nu}g_{\rho\sigma\nu}\sigma^{\rho\sigma}, \\
M &= m + im_5\gamma_5 + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}.
\end{aligned}
$$

- Γ^μ and M are the most general matrices (*e.g.*, m_5).
- SME coefficients: $a_\mu, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\mu, f_\mu, g_{\mu\nu\rho}, H_{\mu\nu}$.

Phenomenology



Experiments and bounds

Experiments (partial list)

- Accelerator/collider.
- Astrophysical observations.
- Birefringence/dispersion.
- Clock-comparison.
- CMB polarization.
- Laboratory gravity tests.
- Matter interferometry.
- Neutrino oscillations.
- Particle *vs.* antiparticle.
- Resonant cavities and lasers.
- Sidereal/annual variations.
- Spin-polarized matter.

No evidence of LV $\Rightarrow$ bounds:

"Data Tables for Lorentz and CPT Violation"
Kostelecký+Russell RMP (2011),
('17 version: arXiv:0801.0287v10)

- $>$ 150 experimental results.
- Best bounds:
  matter $\sim 10^{-34}$ GeV,
  photons $\sim 10^{-43}$ GeV

Gravity SME sector

- Gravity is coupled with SME coefficients (not matter).
- "Minimal" subsector:

$$\begin{aligned}\mathcal{L}_{\mathrm{LV}} &= \sqrt{-g}\, k^{abcd} R_{abcd} \\ &= \sqrt{-g}\left[-uR + s^{ab}R_{ab} + t^{abcd}W_{abcd}\right].\end{aligned}$$

- Decade long puzzle[1]: "the t-puzzle."
- Recently[2] found that t^{abcd} is indeed physical.
- Produces cosm. anisotropies during inflation (tensor modes).
- CMB data (BB angular power spectrum): $t^{0i0j} < 10^{-43}$.
- 29 orders of mag. improvement w.r.t. best bounds on s^{ab}!

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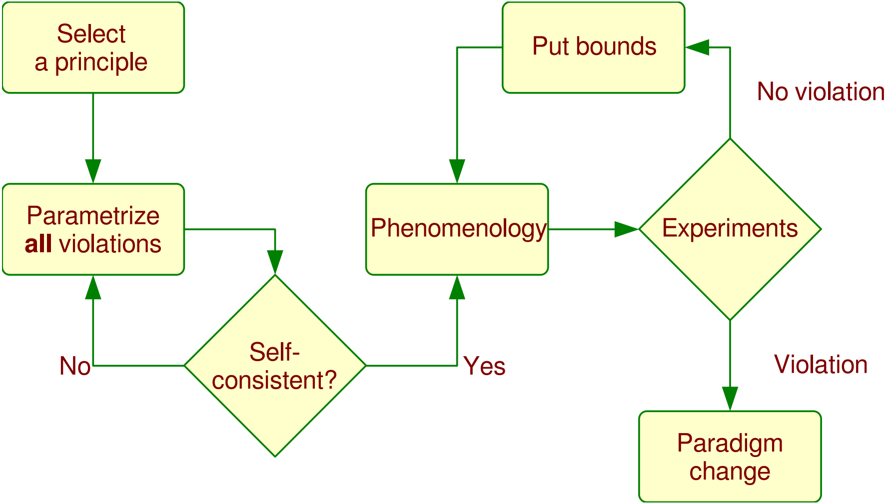
[1] Kostelecký+Bailey PRD 2006
[2] Bonder+León PRD 2017

Self-consistent?



Self-consistency

- Are there theoretical restrictions to rule out LV terms?
- In flat spacetime, few interesting tests.
  - Field redefinitions: Only some linear combinations of the coefficient's components are observable.
- Strong evidence that spacetime is not flat.
- Curved spacetime tests:
  - Field redefinitions.
  - Diffeomorphism invariance.
  - Dirac algorithm and Cauchy problem.
  - Gravitational d.o.f.
  - Spacetime boundaries.

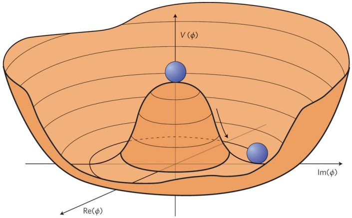
Field redefinitions

- $\psi \to e^{i a_\mu x^\mu} \psi$ shows that $a_\mu \bar{\psi} \gamma^\mu \psi$ is unphysical.
- In flat spacetime, one-to-one correspondence between coordinates and vectors.
- This cannot be done in curved spacetime.
- Less field redefinitions $\Rightarrow$ access more coefficients[1].
- No need for curvature, only nonminkowskian coordinates[2].
- The metric can be redefined $\Rightarrow$ alternative constraints[3].

[1] Kostelecký+Tasson PRD 2011
[2] Bonder PRD 2013
[3] Bonder PRD 2015

Diffeomorphism invariance

- Nondynamical fields break (active) diffeomorphism invariance.
- Thus, $\nabla_a T^{ab} \neq 0$, which goes against the Bianchi identities!
- Position: LV must be spontaneously broken[1].



[1] Kostelecký PRD 2004

Dirac algorithm and Cauchy problem

- Dirac algorithm: Is there a Hamilton density for which the evolution respects the constraints?
- Cauchy problem:
  - Is the evolution uniquely determined by proper initial data?
  - Is the evolution continuous under changes of initial data.
  - Are the effects of modifying the initial data in agreement with spacetime causal structure?

- These conditions are difficult to verify without specifying the coefficients dynamics.

Cauchy problem: concrete model

- Focus on a concrete model[1]:

$$\mathcal{L} = \frac{1}{2} D_\mu \phi D^\mu \phi^* - \frac{m^2}{2} \phi\phi^* - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\kappa}{4} (B_\mu B^\mu - b^2)^2$$

  - Flat spacetime, complex scalar field ϕ (matter), real vector field B^μ.
  - $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $D_\mu \phi = \partial_\mu \phi - ieB_\mu \phi$
    $\Rightarrow \mathcal{L}_{\rm LV} = -B^\mu J_\mu$ and no gauge freedom.
  - Generalization of the Mexican hat potential, its VEV is timelike.
  - e, κ, and b are real positive constants.

- Canonical momenta:

$$\pi^0 = \frac{\delta \mathcal{L}}{\delta \partial_0 B_0} = 0, \quad \pi^i = \frac{\delta \mathcal{L}}{\delta \partial_0 B_i} = B^{i0},$$

$$p = \frac{\delta \mathcal{L}}{\delta \partial_0 \phi} = \frac{1}{2} (\partial_0 \phi^* + ieB_0 \phi^*) = (p^*)^*.$$

[1] Bonder+Escobar PRD 2016

Cauchy problem: concrete model

$$\mathcal{L} = \frac{1}{2} D_\mu \phi D^\mu \phi^* - \frac{m^2}{2} \phi \phi^* - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\kappa}{4} (B_\mu B^\mu - b^2)^2$$

- Two second-class constraints:

$$\begin{aligned} \chi_1 &= \pi^0, \\ \chi_2 &= \partial_i \pi^i - \kappa B_0 (B_\mu B^\mu - b^2) + 2e \mathrm{Im}(\phi p). \end{aligned}$$

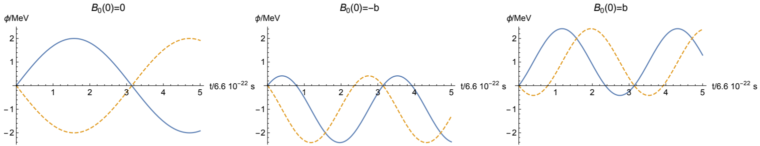
- The Dirac algorithm exhausted without inconsistencies.
- E.o.m. not of the form where one can use the "initial value" theorems.
- D.o.f.: B_i, π^i, ϕ, and p (only this initial data needed) $\Rightarrow$ the initial B_0 obtained through the constraints.
- No unique initial $B_0 \Rightarrow$ ill-posed Cauchy problem!

Cauchy problem: concrete model

- Example (homogeneous): initially $B_i = 0$, $\pi^i = 0$, $\phi = 0$, and $p = a \in \mathbb{C}$.

$$\chi_2 = \left[B_0(0)^2 - b^2\right] B_0(0) = 0 \quad \Rightarrow \quad B_0(0) = b, 0, -b.$$

- Numerically ($\kappa = b/\mathrm{MeV}^2 = e = m/\mathrm{MeV} = \mathrm{Re}(a)/\mathrm{MeV} = \mathrm{Im}(a)/\mathrm{MeV} = 1$):



where the blue (yellow-dotted) line is for $\mathrm{Re}\phi$ ($\mathrm{Im}\phi$).

- ϕ represents matter $\Rightarrow$ physical consequences!

Cauchy problem: concrete model

- Easy fix: change the kinetic term for B_μ... but the Cauchy problem for gravity can be damaged.
- Alternatives:
  - "Only one measurement".... per spatial point (unlike a fundamental constant).
  - Consider B_0 as a standard d.o.f. (*i.e.*, naive application of Lagrange's formalism $\Rightarrow$ inequivalent quantizations?, discrete number of d.o.f.).
  - Construct a criteria to choose a special B_0 (*e.g.*, initial energy, but there are degeneracies).
- Longterm goal: study if we can rule out spontaneous LV.

Gravitational degrees of freedom

- Palatini vs. conventional
  - For the minimal gravitational LV, the standard and Palatini approaches are equivalent[1].
  - More general field redefinitions, no practical applications!
  - For nonminimal LV, these approaches are inequivalent.
- Boundaries
  - In the phenomenological applications of LV, spacetime is conformally flat, which has boundaries.
  - For the minimal gravitational action, add[2]

$$\Delta S_{\rm LV} = \pm 2 \int_{\rm boundary} d^3x \sqrt{|h|} n_\mu n_\sigma k^{\mu\nu\rho\sigma} K_{\nu\rho}.$$

  - Tricky to find $\Delta S_{\rm LV}$ for the nonminimal part!

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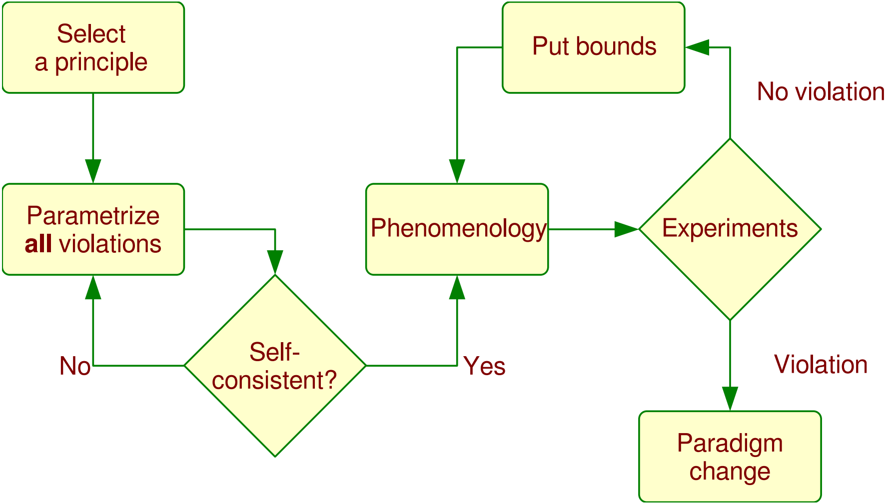
[1] Bonder PRD 2015

[2] Bonder PRD 2015

Conclusions

- Looking for empirical clues of new physics could play an important role towards QG.
- This must be done systematically: with generality and checking the self-consistency.
- This type of program has been applied mainly for LV.
- EFT provides the general parametrization.
- Such a parametrization allows us to test LV experimentally and theoretically.
- New interesting phenomenological connections with cosmological observations.
- Several theoretical restrictions, mainly in curved spacetime.

Paradigm change



Gibbons-Hawking term

- In the minimal gravitational LV action-variation:

$$\begin{aligned}\delta S &\supset \frac{1}{2\kappa}\int_M d^4x\sqrt{-g}(g^{ca}\delta^b_d + k^{abc}{}_d)\delta R_{abc}{}^d \\ &= \frac{1}{\kappa}\int_M d^4x\sqrt{-g}(\nabla_c\nabla_d k^{cabd})\delta g_{ab} \\ &\quad +\frac{1}{\kappa}\int_{\partial M} d^3x\sqrt{|h|}n_c(2g^{a[c}g^{b]d} + k^{cabd})\nabla_d\delta g_{ab}\end{aligned}$$

- In ∂M: $\delta g_{ab} = 0$ (and $\delta h_{ab} = \delta n^a = 0$) but $n^c\nabla_c\delta g_{ab} \neq 0$.
- $K_{ab} = h^c_a\nabla_c n_b \Rightarrow \delta K_{ab} = -h^c_a n_d\delta C_{cb}{}^d = \frac{1}{2}h^c_a n^d\nabla_d\delta g_{bc} \Rightarrow$

$$n_c(2g^{a[c}g^{b]d}+k^{cabd})\nabla_d\delta g_{ab} = -\delta[\left(2h^{ab} \pm 2n_c n_d k^{cabd}\right)K_{ab}],$$

- To cancel the problematic term:

$$\Delta S = \frac{1}{\kappa}\int_{\partial M} d^3x\sqrt{|h|}\left(2h^{bc} \pm 2n_a n_d k^{abcd}\right)K_{bc}.$$

Variation under diffeomorphisms

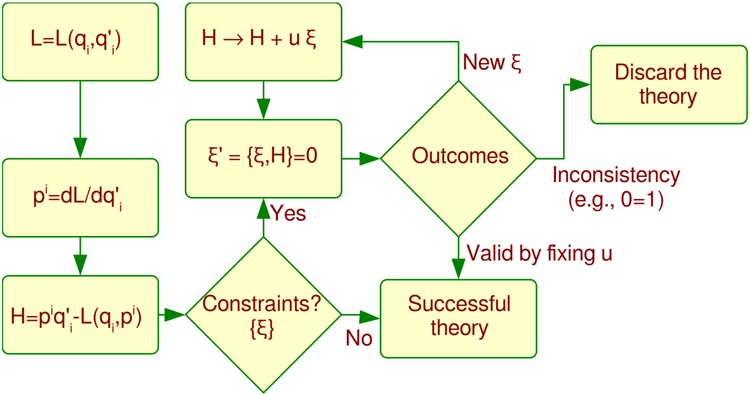
- Nongravitational LV: $S = \int d^4x\sqrt{-g}R + 2\kappa S_{\mathrm{m}}(g, \phi; k)$.
- Under a diffeo. assoc. with any ξ^a (of compact sup.):

$$\begin{aligned}
\delta S &= \int d^4x \left(\frac{\delta\sqrt{-g}R}{\delta g^{ab}}\delta g^{ab} + 2\kappa\frac{\delta\mathcal{L}_{\mathrm{m}}}{\delta g^{ab}}\delta g^{ab} + 2\kappa\frac{\delta\mathcal{L}_{\mathrm{EH}}}{\delta\phi}\delta\phi \right) \\
&= \int d^4x \left(-G_{ab} + \kappa T_{ab}\right)\left(-2\nabla^{(a}\xi^{b)}\right) \\
&= 2\int d^4x \left(-\nabla^a G_{ab} + \kappa\nabla^a T_{ab}\right)\xi^b \\
&= 2\kappa\int d^4x\xi^b\nabla^a T_{ab},
\end{aligned}$$

where use that the fields ϕ satisfy their e.o.m.,
$\delta g^{ab} = \mathcal{L}_\xi g^{ab} = -2\nabla^{(a}\xi^{b)}$, and the Bianchi identity.
- Hence, $\delta S = 0$ if and only if $\nabla_a T^{ab} = 0$.

Dirac method

- Dirac's algorithm: method to construct *the* Hamiltonian.



- May reveal inconsistencies (example: $L(q, \dot{q}) = q$).

Cauchy theorems

- Cauchy-Kowalewski requires analytic initial data, which damages causality.

Theorem

(M, g_{ab}) globally hyperbolic, ∇_a any derivative operator. The following system of n linear equations for n unknown functions $\Psi_1, \ldots, \Psi_n$

$$g^{ab}\nabla_a\nabla_b\Psi_i + A^a_{ij}\nabla_a\Psi_j + B_{ij}\Psi_j + C_i = 0,$$

where A^a_{ij}, B_{ij}, C_i are smooth vector/scalar fields, has a well-posed Cauchy problem.

- There are more general theorems[1].
- Most relevant: form of the second-derivative term.

[1] Wald's GR book