Inhomogeneous Cosmological models, what are they good for?

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If the ΛCDM paradigm model fits observations so well, then why bother looking for alternatives ?

- Because "dark matter & "dark energy" are "black boxes" whose detection has been very elusive.

- Because what we have is just

GR+ FLRW + linear perturbations + CDM + Lambda → Observations are well fit

The converse of this implication is NOT (necessarily) true:
fitting observations DOES NOT IMPLY the ΛCDM model

as long as we don't know the fundamental nature of "DM" & "DE" there is justification in trying to fit observations with other models or even other gravity theories

We live in the midst of a scientific controversy:

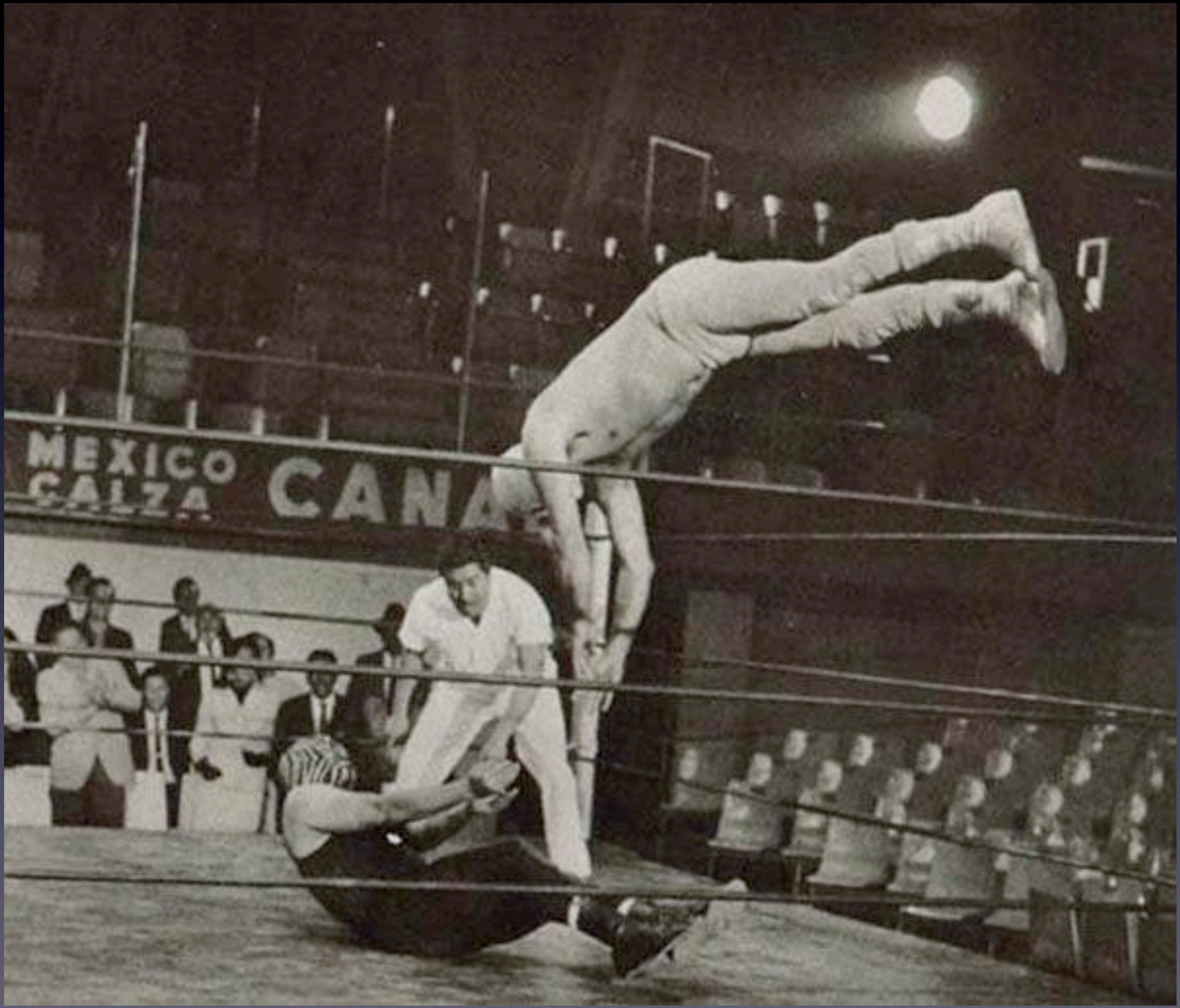
The Orthodoxy says

Your gravity theory is wrong Idiot !!

CDM SUCKS you jerk !!

There's no DARK ENERGY Stupid !!

HOPEFULY, there's no need to solve this controversy in the ring !



A "conservative" set of assumptions to fit cosmic observations outside the Λ-CDM model is to

Keep GR as gravity theory

CDM (or some form of Dark Matter) exists

BUT assume that

$\Lambda = 0$ *(or there is no Dark Energy)*

→ *FLRW model with linear perturbations DOES NOT provide an appropriate large scale description of Cosmic Dynamics*

→ *The Universe should be inhomogeneous at large scales*

→ *Observations should be fit by inhomogeneous GR models WITHOUT assuming $\Lambda > 0$ or any form of DE*

This set of assumptions MUST BE TESTED

How to construct Inhomogeneous Cosmological models in general

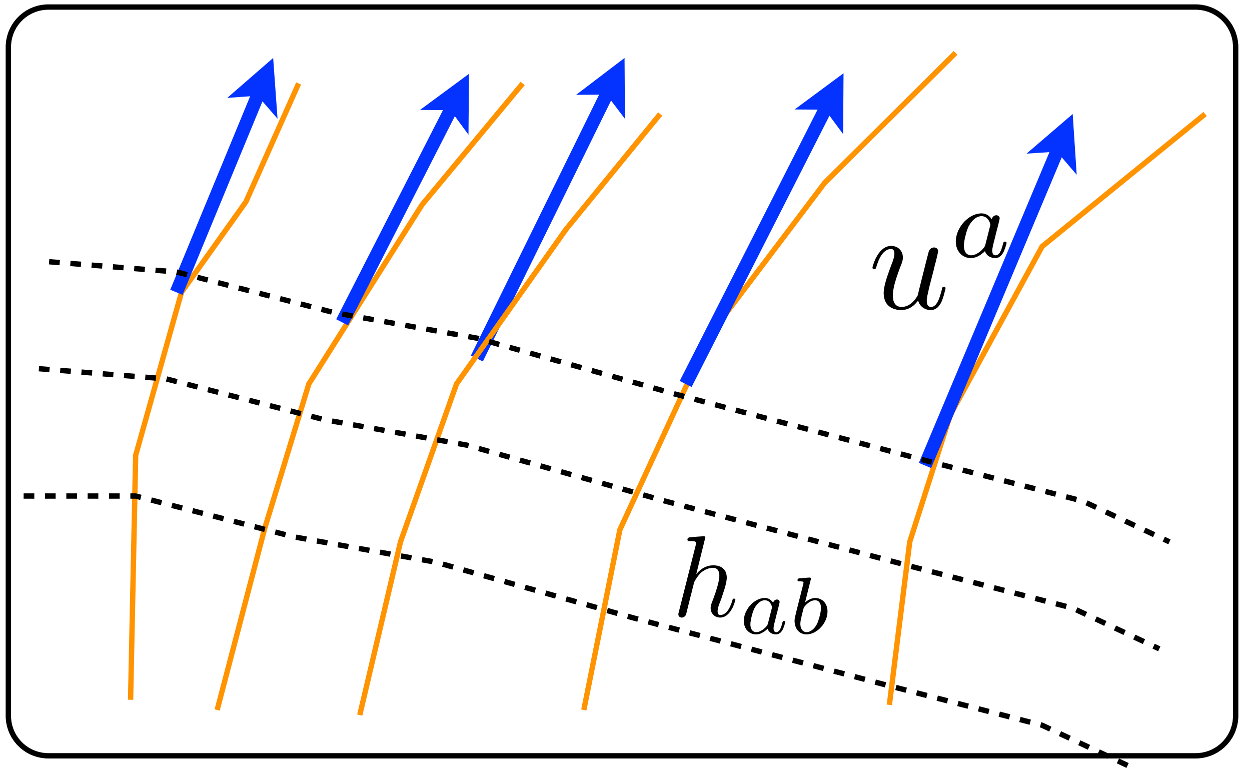
Take a spacetime manifold $(\mathcal{M}, g)$

- **satisfies Einstein's equations**

$$G^{ab} = 8\pi T^{ab} \qquad \nabla_b T^{ab} = 0$$

- **Admits a 4-velocity field** u^a

Define a class of Cosmic Observers



Comoving with 4-velocity u^a

"Time derivative" is $\frac{d}{d\tau} = u^a \nabla_a$

Observers define spatial metric

$$h_{ab} = g_{ab} + u_a u_b$$

"Spatial gradients" are

$$\tilde{\nabla}_a = h^b_a \nabla_b$$

General cosmological model is too difficult. We need further assumptions for a CDM dominated universe

- Negligible energy flux and viscous stress $\Pi_{ab} \approx 0, \quad q_a \approx 0,$
- Negligible pressure $\rho \approx mnc^2, \quad p \approx mn\langle v^2 \rangle \ll \rho$
- Negligible pressure gradients $h_a^b \nabla_b(p) \ll h_a^b \nabla_b(\rho) \quad \Rightarrow \quad \dot{u}_a \approx 0$
- Negligible vorticity (rotation) $\omega_{ab} \approx 0$
- Negligible vector & tensor modes (magnetic fields & gravitational waves) $H_{ab} \approx 0$ **Weyl tensor is electric**

→ **Dynamics reduces to scalar modes**

Inhomogeneous dust universes: dynamical system on

$\rho, \quad \mathcal{H} = \frac{\Theta}{3},$ CDM density & Hubble scalar (common to FLRW)

$E_{ab}, \quad \sigma_{ab},$ Electric Weyl & shear tensors (absent in FLRW) -- deviation from homogeneity

→ **FLRW models follow if** $E_{ab} = \sigma_{ab} = 0$

Dynamical System on $\rho, \ \mathcal{H}$

Hierarchy of known exact solutions

Szekeres models: non-spherical (dipole-like)

$$ds^2 = -dt^2 + a^2\left[\frac{\Gamma^2 dr^2}{1-k_0\, r^2} + \frac{r^2\left(dx^2+dy^2\right)}{F^2}\right]$$

As LTB $a = a(t,r)$ but $\Gamma = \Gamma(t,r,x,y),\ F = F(r,x,y)$

all quantities depend on (t,r,x,y) in the form $A = A_1(t,r) + A_2(t,r,x,y)$

Remove Spherical symmetry

LTB models: spherical inhomogeneity

$$ds^2 = -dt^2 + a^2\left[\frac{\Gamma^2 dr^2}{1-k_0\, r^2} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right)\right]$$

two scale factors

$a = a(t,r),\ \Gamma = \Gamma(t,r)$

all quantities depend on (t,r)

$$\rho(t,r) = \frac{\rho_0(r)}{a^3\Gamma},\ \mathcal{H}(t,r) = \frac{\dot a}{a} + \frac{\dot\Gamma}{3\Gamma}$$

Spherical inhomogeneity

FLRW models: homogeneous

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-k_0\, r^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2\right],$$

one scale factor

$a = a(t)$

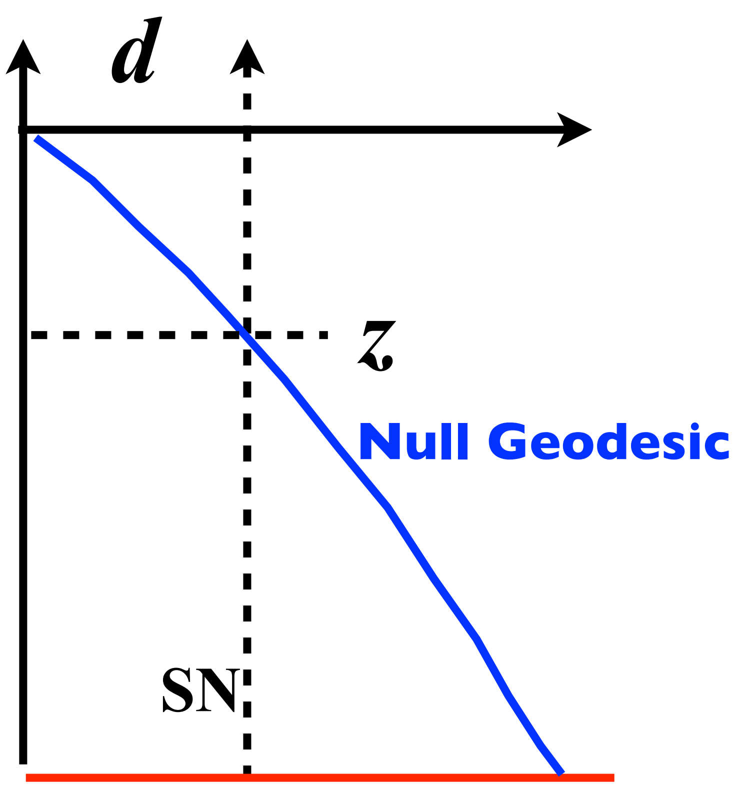
all quantities are time dependent

$\rho = \rho(t) = \rho_0 a^{-3}, \qquad \mathcal{H} = \mathcal{H}(t) = \dot a/a$

Why inhomogeneous models with $\Lambda = 0$ may fit cosmic observations?

Because large scale observations is information transmitted by NULL GEODESICS through our past light cone, and ALL the latter is very different for inhomogeneous models

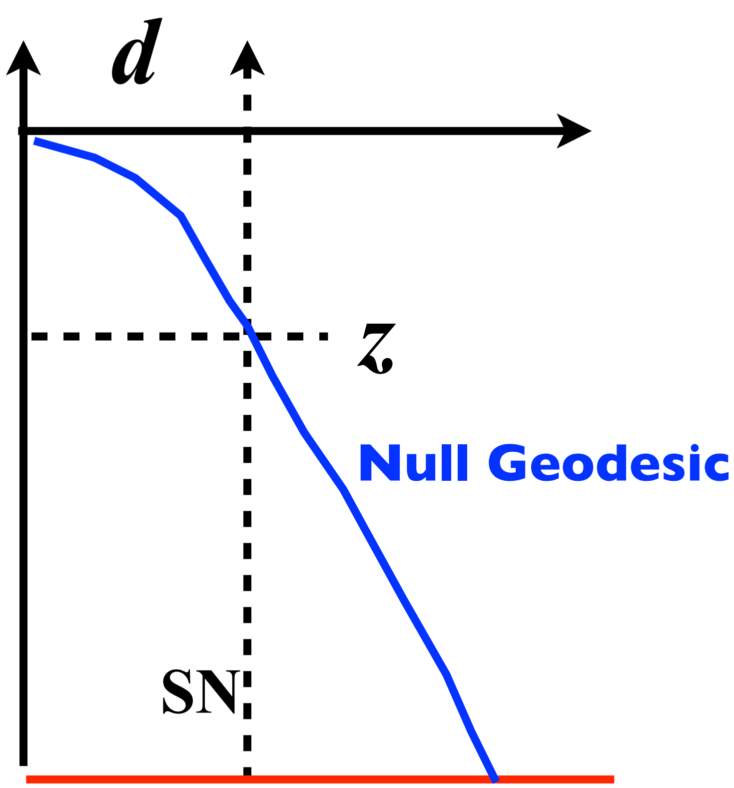
FLRW model



Relation $d = d(z)$ only fits data if $\Lambda > 0$ (accelerated expansion)

Free parameters $H_0,\ \Omega_0^m,\ \Omega_0^\Lambda$

LTB model with $\Lambda = 0$



Relation $d = d(z)$ may fit data with $\Lambda = 0$ for certain density profiles (voids)

Free parameters $H_0(r),\ \Omega_0^m(r),\ \Omega_0^K(r)$

The Hubble diagram & z-distance module relation for single Gpc size void:

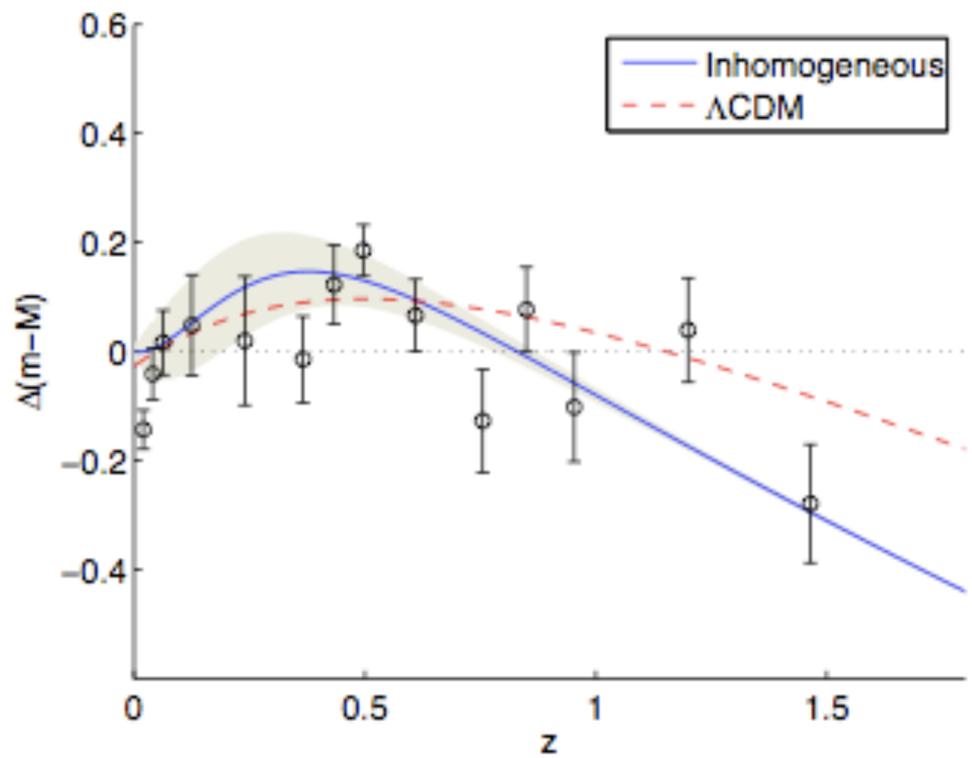


FIG. 2: The average distance modulus for an off-center observer in our model. The shaded area represents the RMS deviation from the average across the sky. The data points and error bars are binned data from the Riess et al. Gold Set, while the red dashed line is the corresponding best-fit ΛCDM model.

$$m - M = 5 \log D_L(z)$$

$$D_L(z) \approx \left(\frac{dD_L}{dz}\right)_0 z + \frac{1}{2}\left(\frac{d^2 D_L}{dz^2}\right)_0 z^2$$

Compare coefficients: BOTH FIT

ΛCDM

$$\left(\frac{dD_L}{dz}\right)_0 = \frac{c}{H_0}$$

$$\frac{1}{2}\left(\frac{d^2 D_L}{dz^2}\right)_0 = \frac{c}{4H_0}\left(2 - \Omega_m + 2\Omega_\Lambda\right)$$

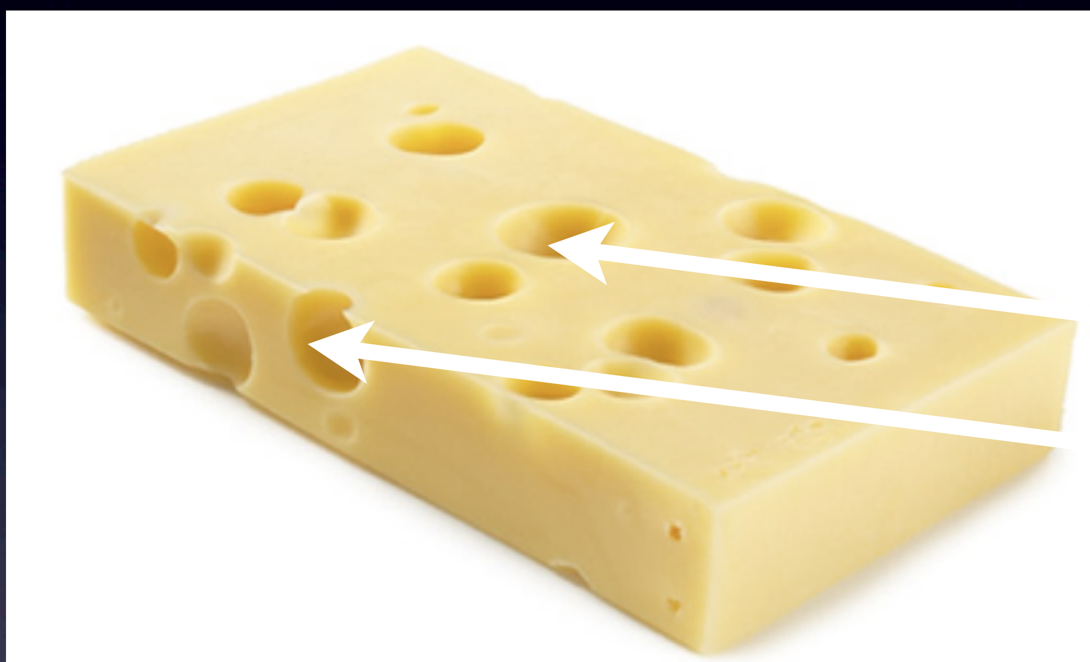
LTB with $\Lambda = 0$

$$\left(\frac{dD_L}{dz}\right)_0 = \frac{c}{H_0}$$

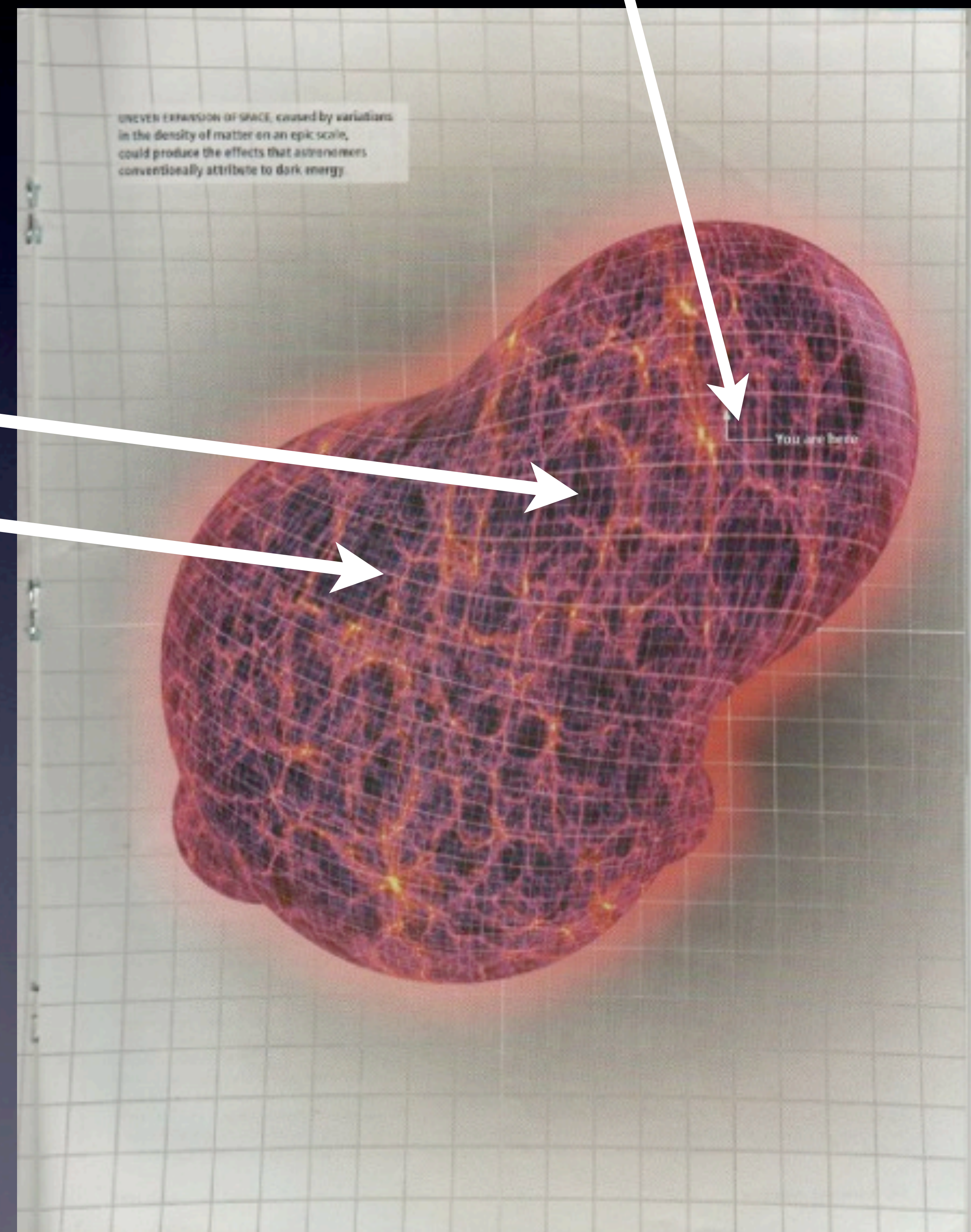
$$\frac{1}{2}\left(\frac{d^2 D_L}{dz^2}\right)_0 = \frac{c}{4H_0}\left[1 + f(\Omega_0^m(r), \Omega_0^k(r))\right]$$

“Swiss Cheese” model: simple pattern of “distributed” inhomogeneities in the Universe

We could be inside of one of these void regions



The cheese holes are the void regions



How to make Swiss cheese models ?

Represent this


[THE BIG PICTURE]

A Special Place for Us

In his *Hitchhiker's Guide to the Galaxy* series of novels, Douglas Adams imagines a torture device that drives people insane by showing them the utter insignificance of their place in the universe. One would-be victim emerges unscathed when it turns out that the universe does, in fact, revolve around him. In a case of life imitating art, many cosmologists are investigating whether our planet indeed has a special place within the grand scheme of things.

HOMOGENEOUS UNIVERSE: OUR LOCATION IS TYPICAL
In the standard view, galaxies are lined up in a spidery pattern, but overall space looks much the same everywhere, and Earth's position is nothing special.

INHOMOGENEOUS UNIVERSE: OUR LOCATION IS SPECIAL
Alternatively, the density of matter could vary on large scales, and Earth may lie at or near the center of a relatively less dense region, or void.

Copernicus principle with a larger homogeneity scale

by this simple model

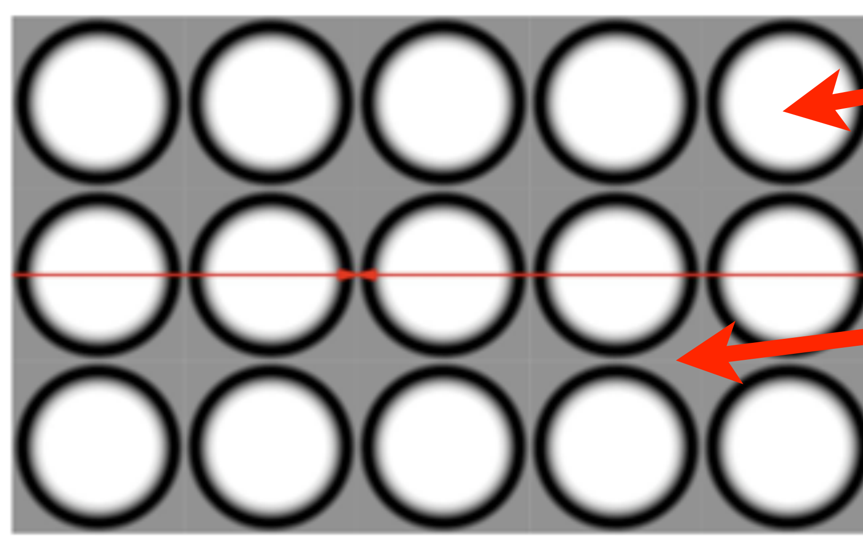
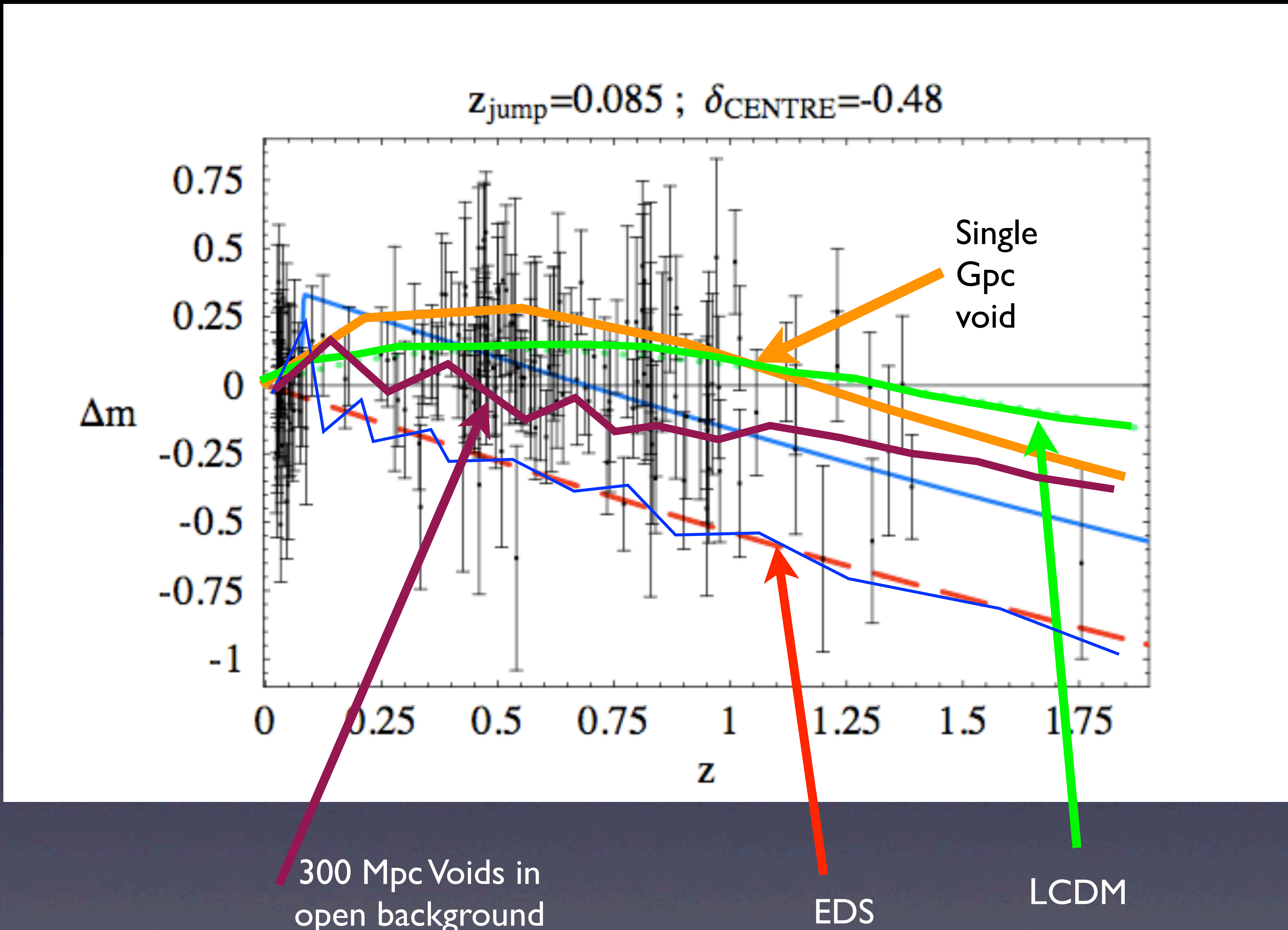


FIG. 2: Sketch of our model. The shading mimics the initial density profile: darker shading implies larger denser. The uniform gray is the FRW cheese. The photons pass through the holes as shown by the arrows and are revealed by the observer placed in the cheese.

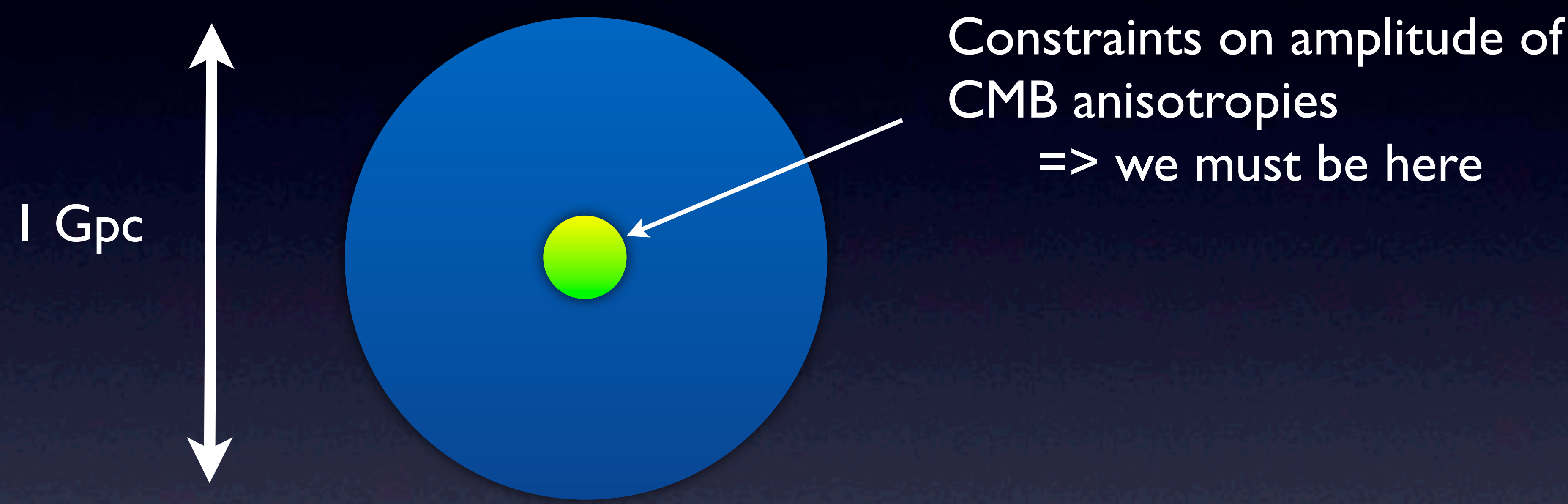
Spherically symmetric dust underdensity (LTB)

The "cheese" is homogeneous dust (FLRW)



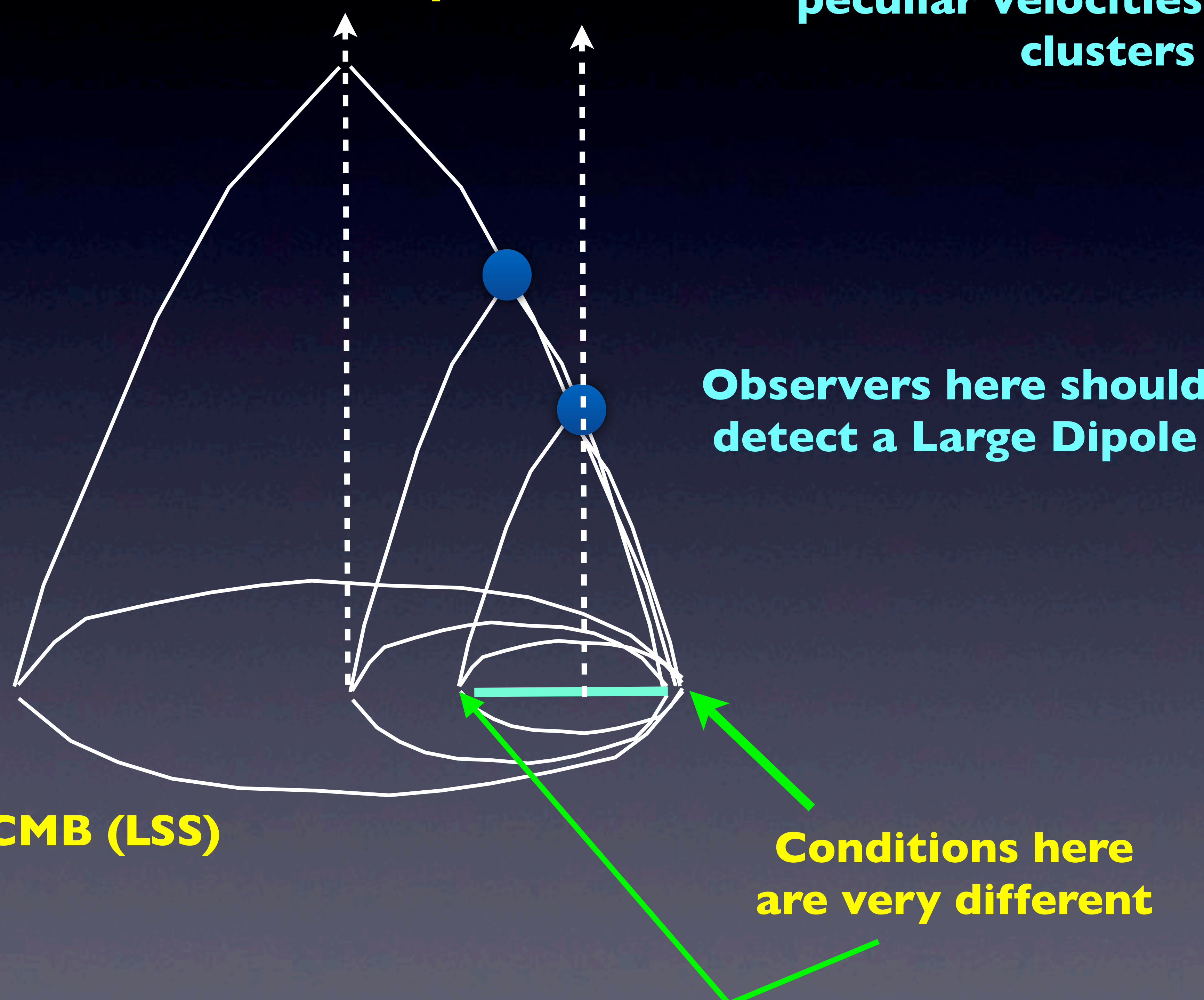
Only Gpc size voids seem to fit SN at higher z

Spherical inhomogeneity is problematic & restrictive:
CMB is almost isotropic, so fitting it with an LTB model requires being "near" the center of the void (fine tuning).



Departure from spherical symmetry suggests that this "center problem" can be removed (or made less binding).

The Kinematic Sunyaev Zeldovich effect



Gpc size Spherical Voids:

Single void occupies all observable Universe [check]

Must comply with several observational tests:

- SN Ia
- CMB amplitude & multipoles & BAO
- Initial conditions (LSS) compatible with inflation
- age constraints & H0 measurement
- kinematic S-Z
- etc,

Not Possible !!

LTB models are too simple
They lack dynamical freedom

How stringent is the KSZ effect?

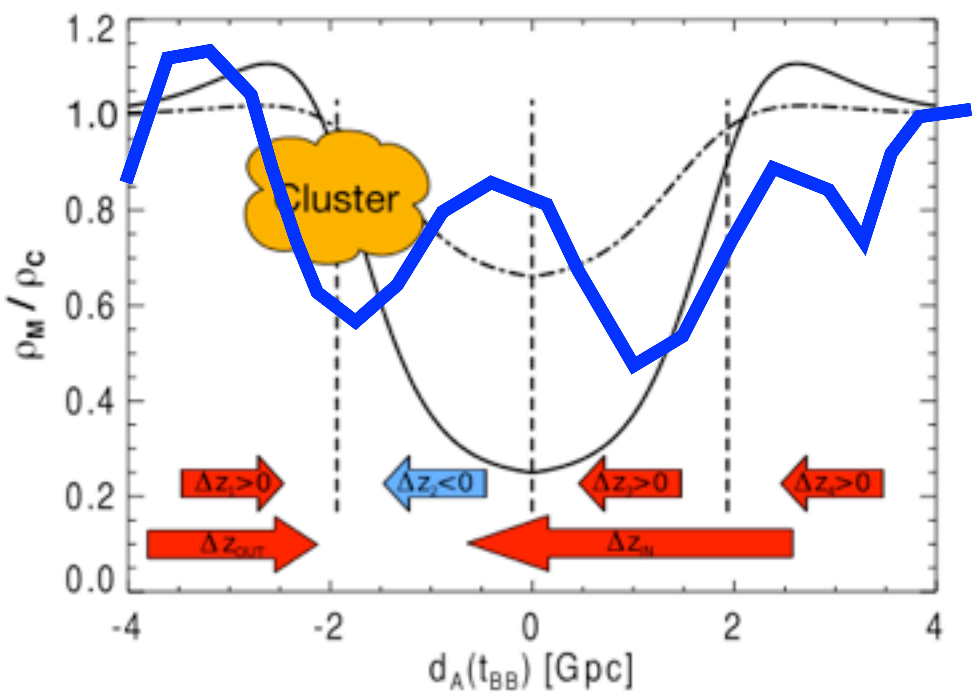


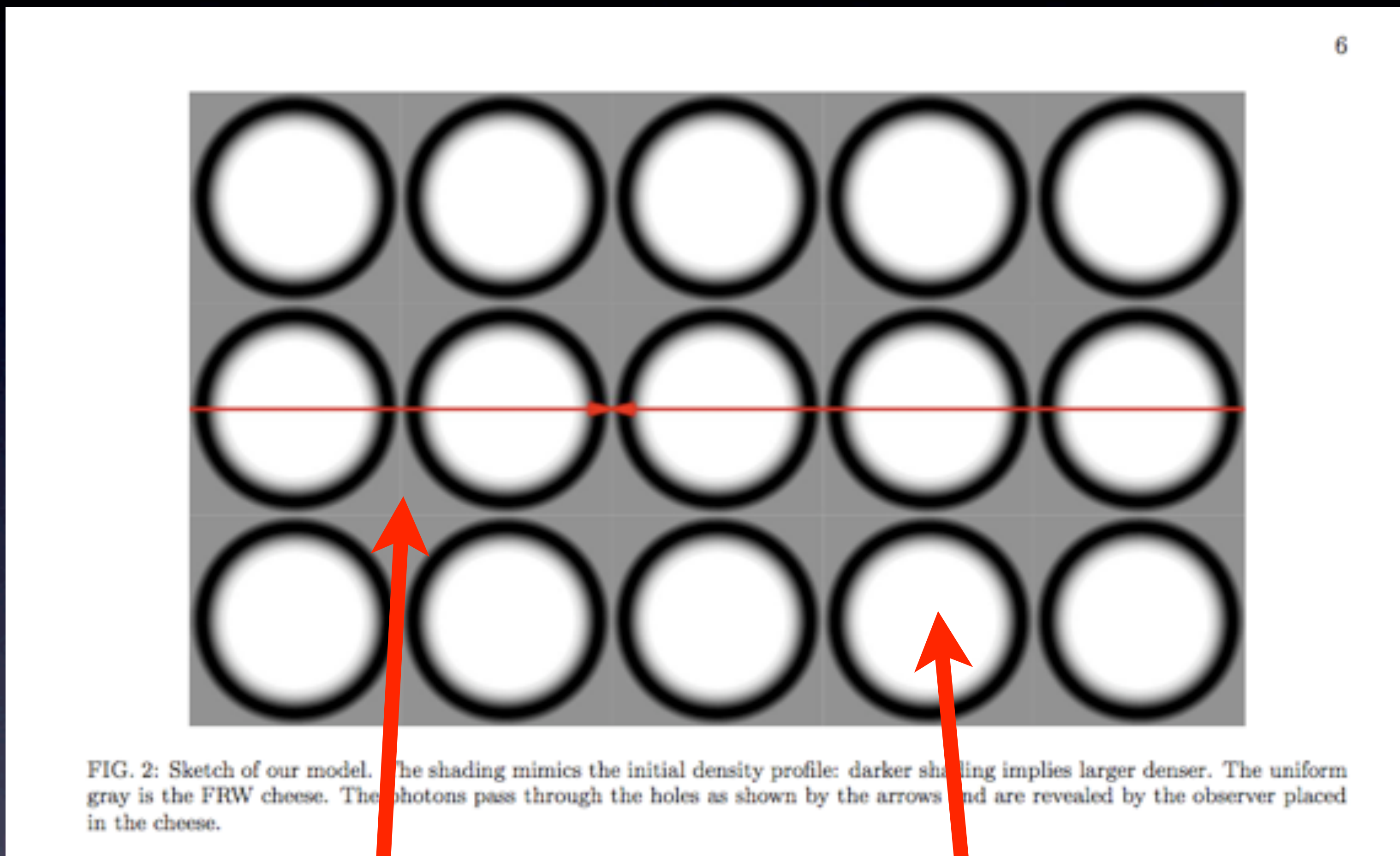
Figure 1. An off-centre cluster of galaxies in a void will "observe" CMB photons coming from the last scattering surface from all directions. Due to the higher expansion rate inside the void, photons arriving through the centre (from the right in the figure) will have a larger redshift ($\Delta z_{\rm in}$), than photons arriving directly from the LSS (left, with $\Delta z_{\rm out}$). There is a subdominant effect due to the time-dependent density profile (the solid line corresponds to the current time, while the dot-dashed line to one tenth of the present time). With a larger underdensity at later times, we have $\Delta z_1 > \Delta z_4$, and $\Delta z_2 + \Delta z_3 < 0$, giving an overall difference $\Delta z_1 > \Delta z_2 + \Delta z_3 + \Delta z_4$ or, equivalently, a subdominant dipole with a blueshift towards the centre of the void. The overall effect is a blueshift away from the centre.

Looking the void in the eyes - the kSZ effect in LTB models, Juan Garcıa-Bellido & Troels Haugbolle, JCAP, [arXiv:0807.1326]

See also
The kSZ effect as a test of general radial inhomogeneity in LTB cosmology, Philip Bull, Timothy Clifton, Pedro G. Ferreira, Phys. Rev. D 85, 024002 (2012), [arXiv: 1108.2222v3 [astro-ph.CO]]

It has been tested ONLY on spherical LTB models: it does NOT rule out general inhomogneity (more work needed)

So, let us go beyond spherical voids

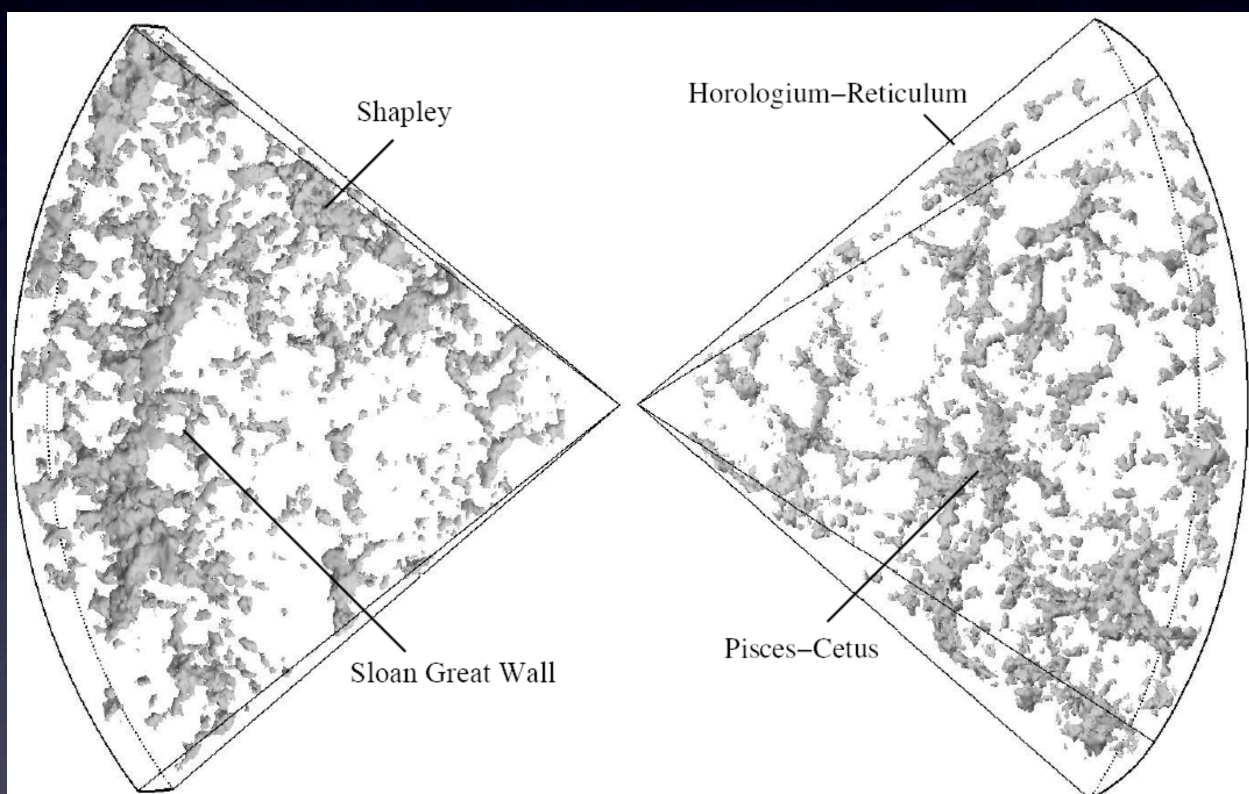


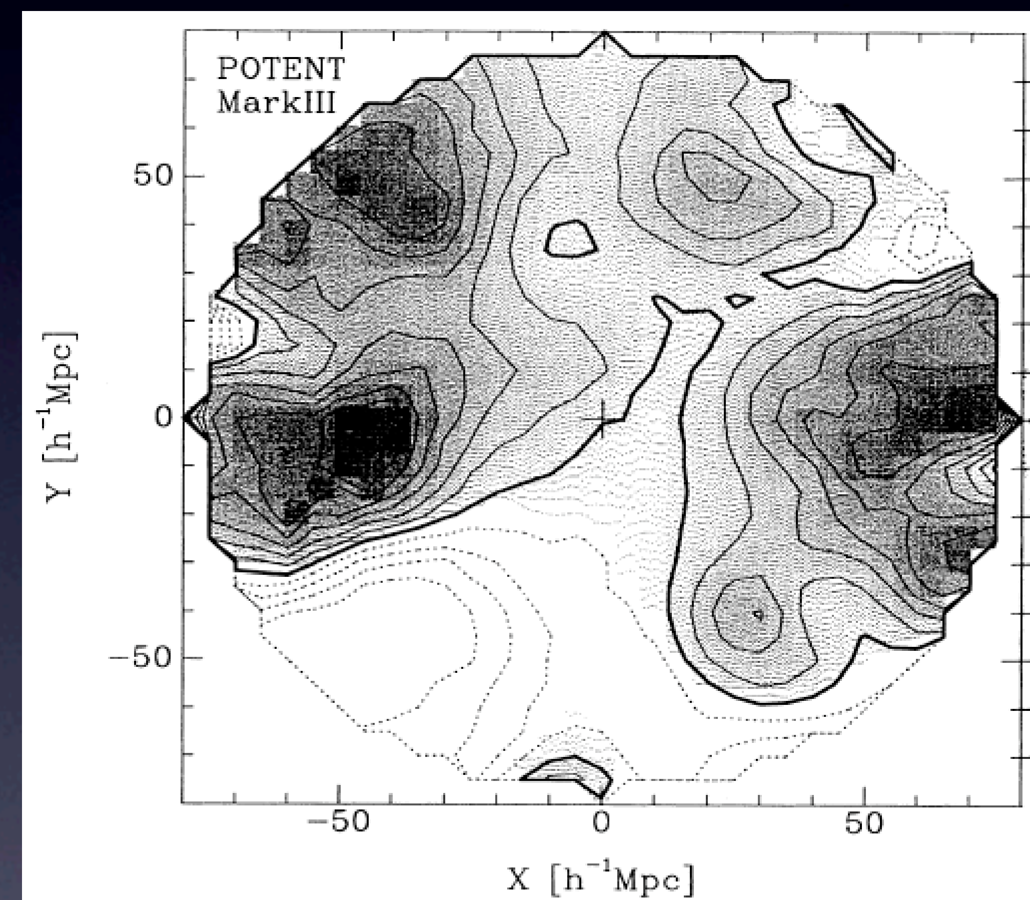
A Swiss cheese model, but the "inside" of the holes is no longer spherically symmetric

The "cheese" is homogeneous dust (FLRW)

Dust underdensities (voids) that are NOT spherically symmetric (Szekeres)

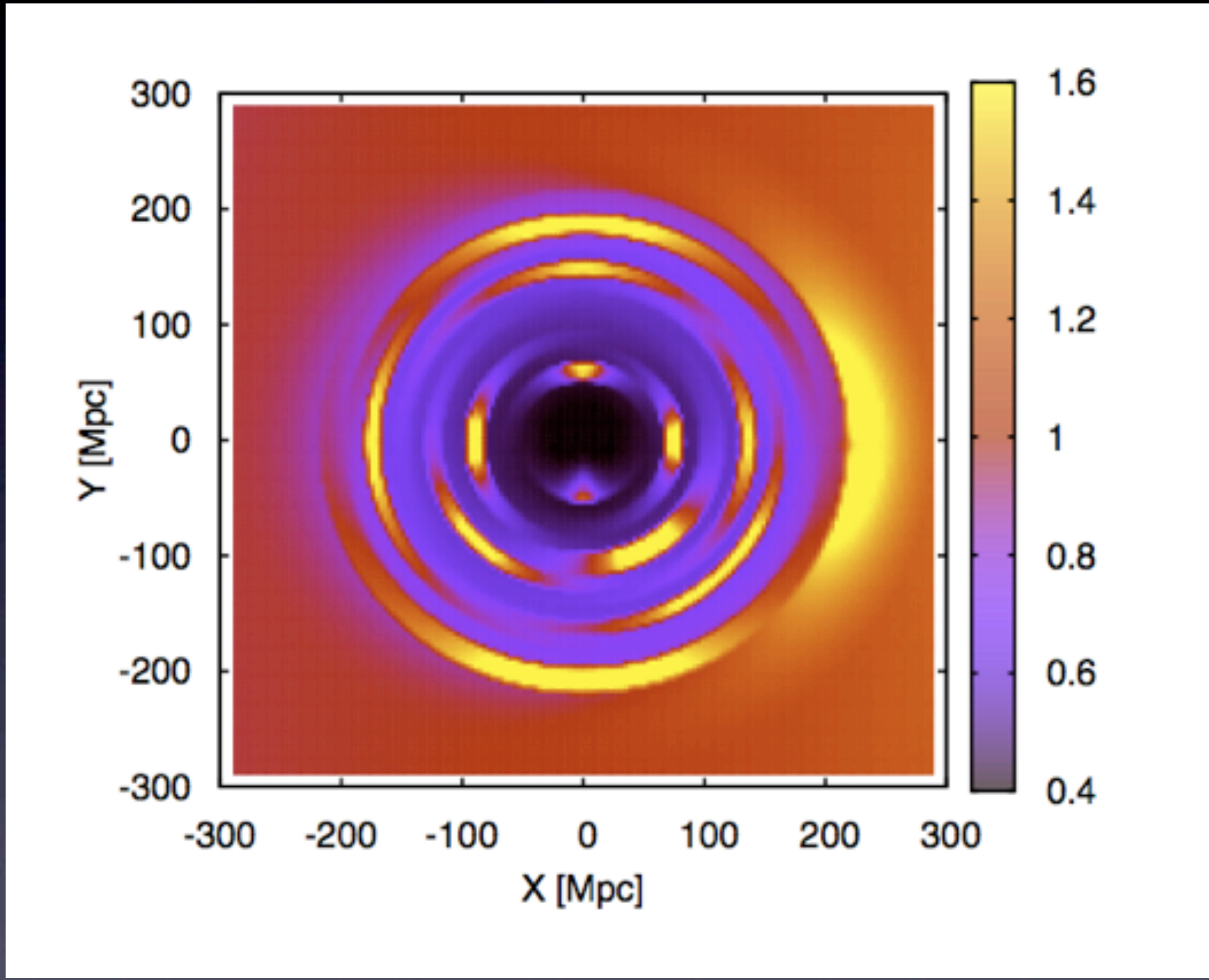
Our cosmography at scales < 300 Mpc is obviously NOT spherically symmetric !!!





We try to model this Cosmography with the Szekeres solution

K. Bolejko & R. A. Sussman, Cosmic spherical void via coarse-graining and averaging non-spherical structures, Physics Letters B 697 (2011) 265-27, arXiv:1008.3420



Cross section (tessellation) of the Szekeres density at the "equator"

Coarse-graining cosmic structure by Szekeres solutions

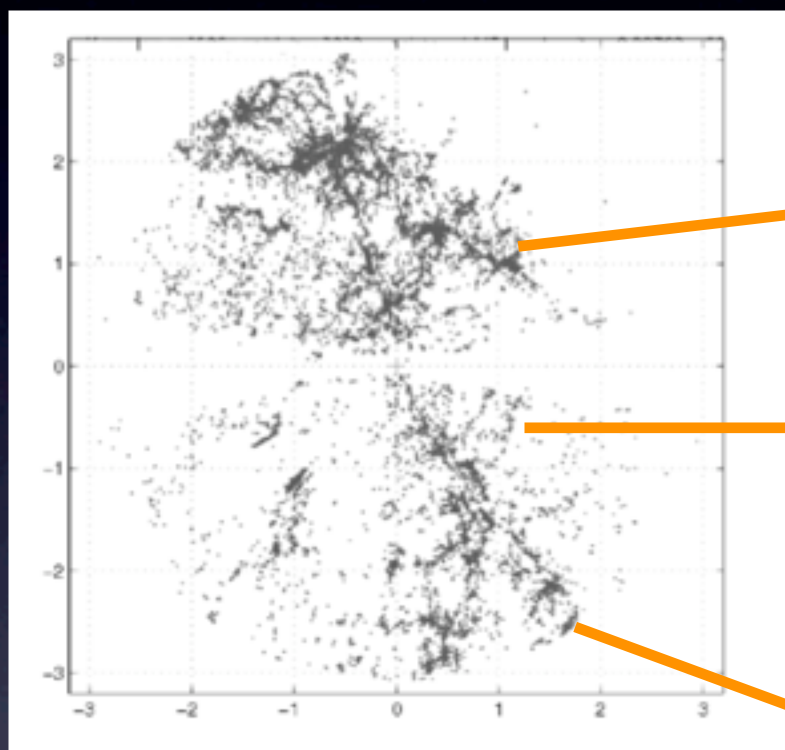
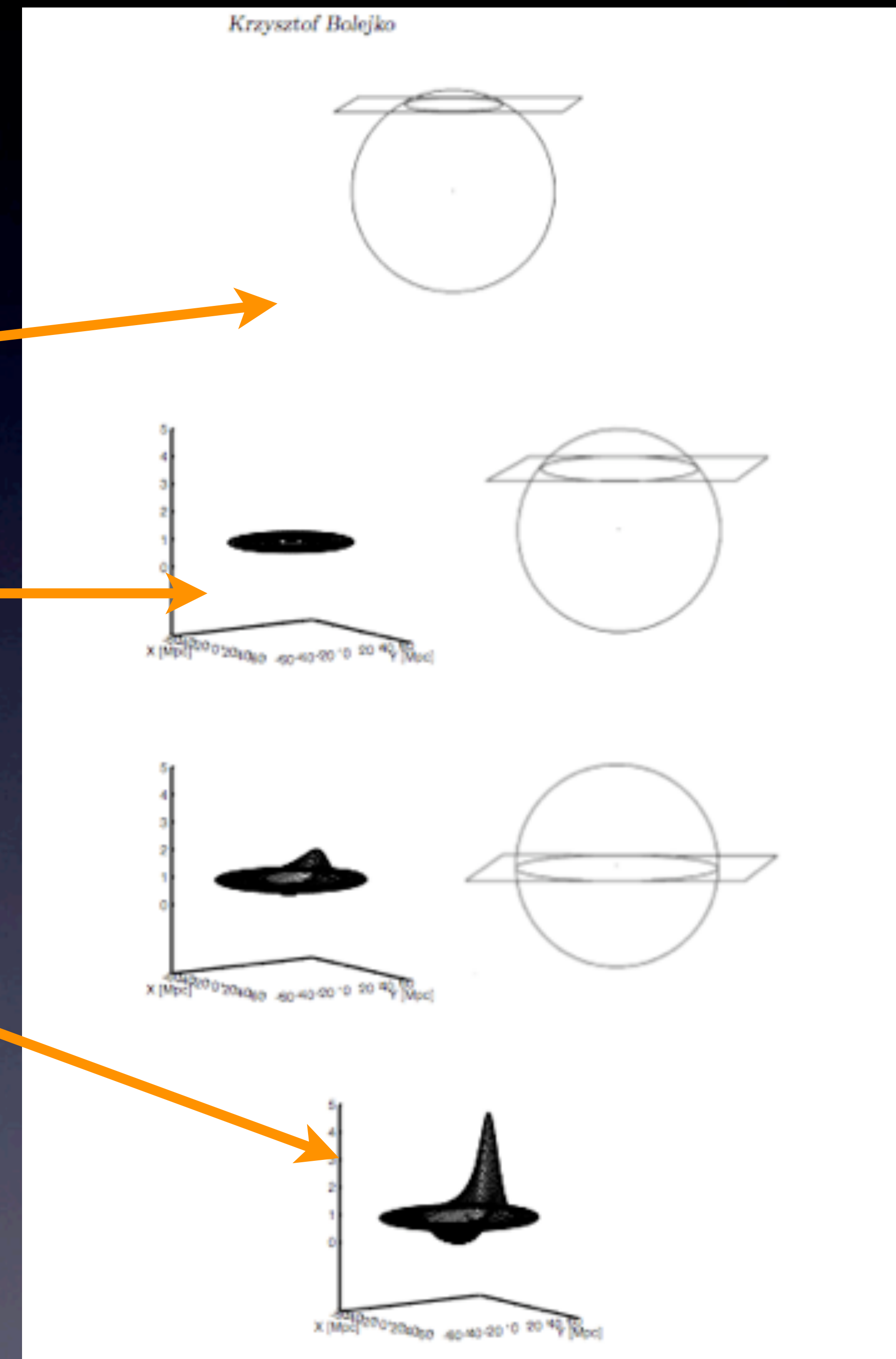


Figure 1. The galaxy distribution in a slice 1600 km/s thick and 32000 km/s square. The Zone of Avoidance is clearly visible in the middle of the picture. Since the data are from different redshift surveys, some structures are well mapped, while other regions are not. Still there are regions from which there is no data at all.



K. Bolejko
Structure formation in the quasispherical Szekeres model
Phys.Rev.D73:123508,2006

Models of voids & overdensties that "interact"

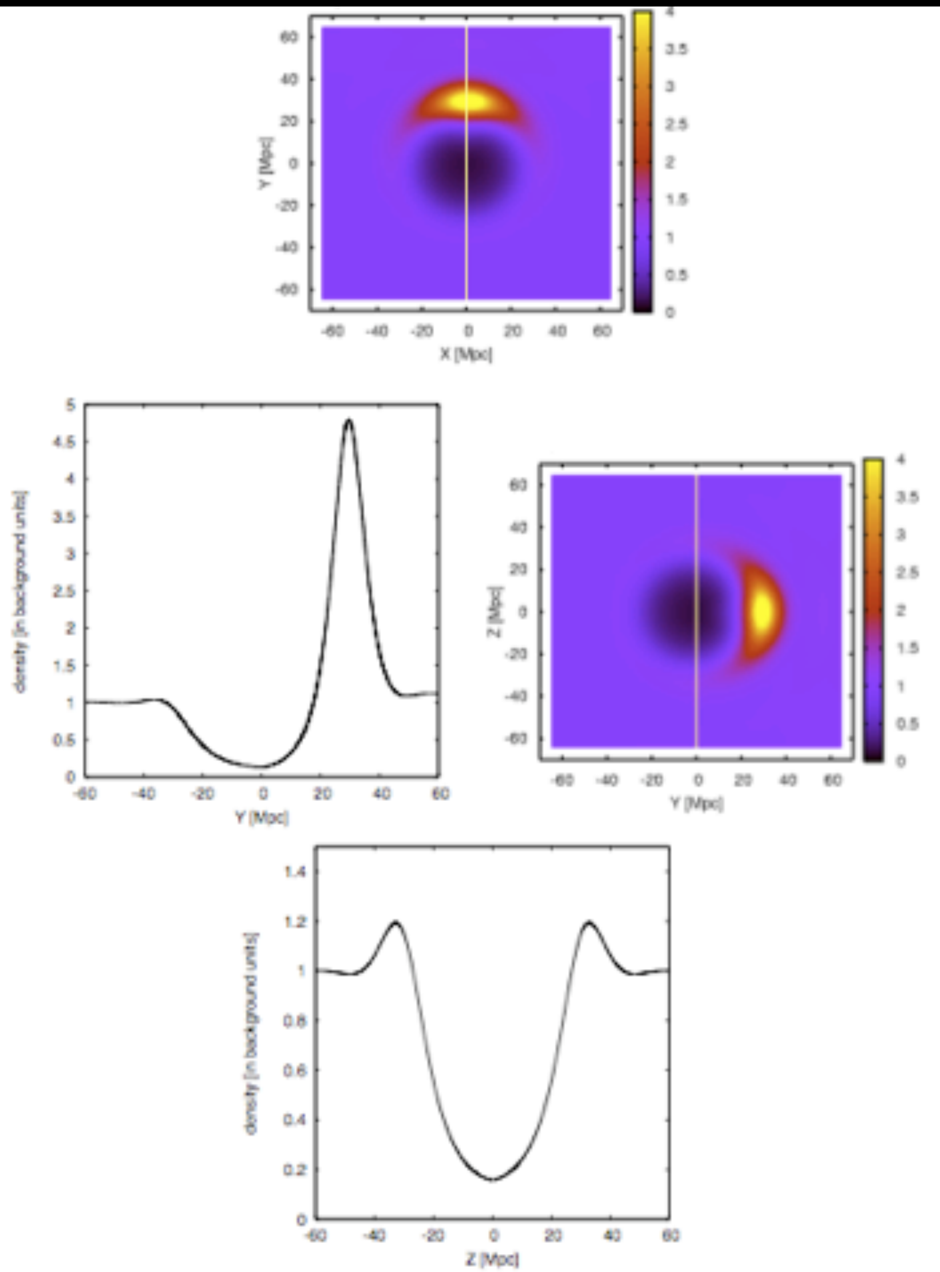


Figure 4. Density distribution in the considered structure. Upper left panel presents colour coded density distribution of the equatorial cross section (see Fig. 3, bottom panels). Lower left panel presents the vertical cross–section of $X = 0$, through the considered model. The yellow lines correspond to the density profiles, which are presented on the right side. For detailed description see Sec. 6.

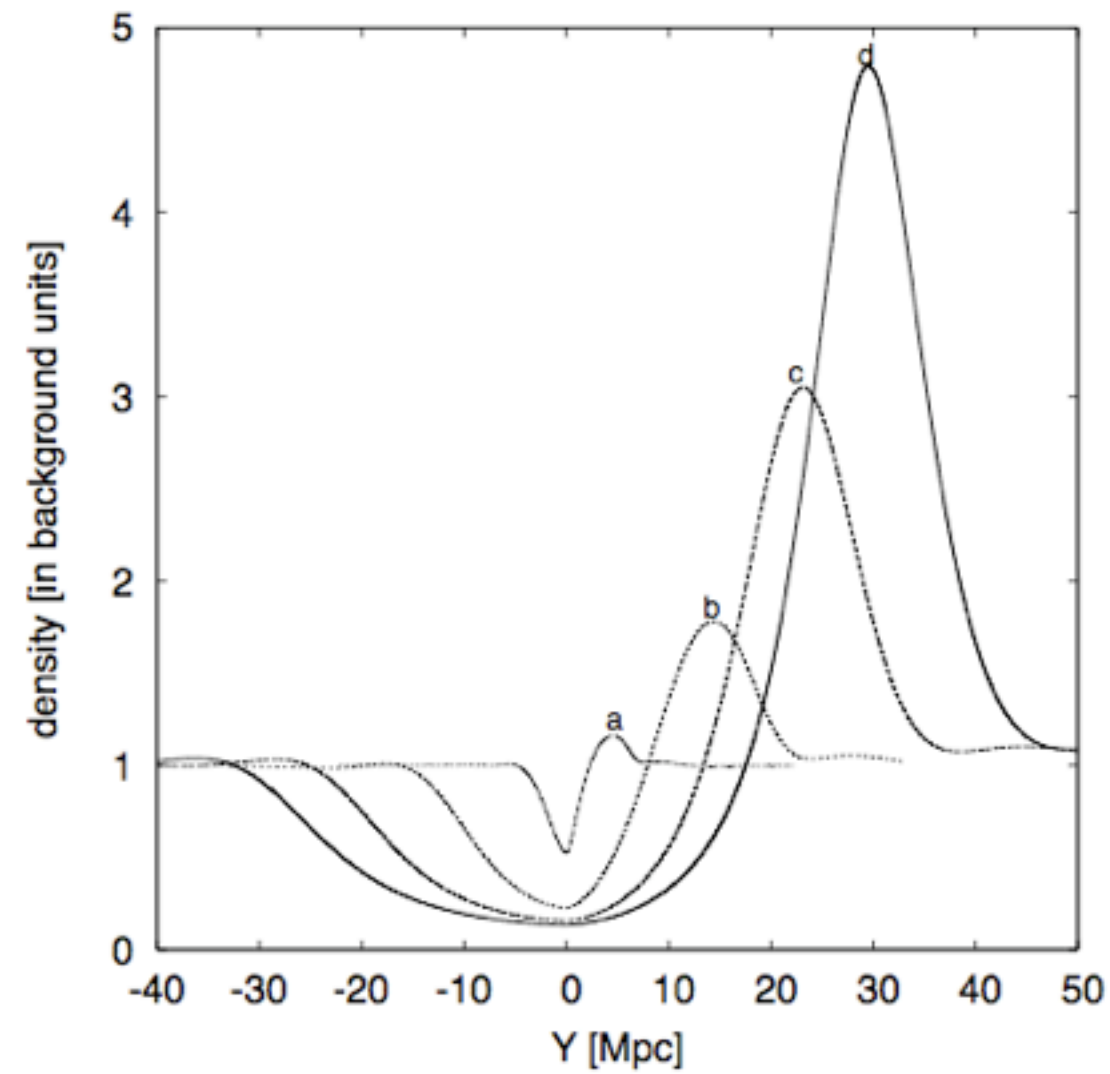
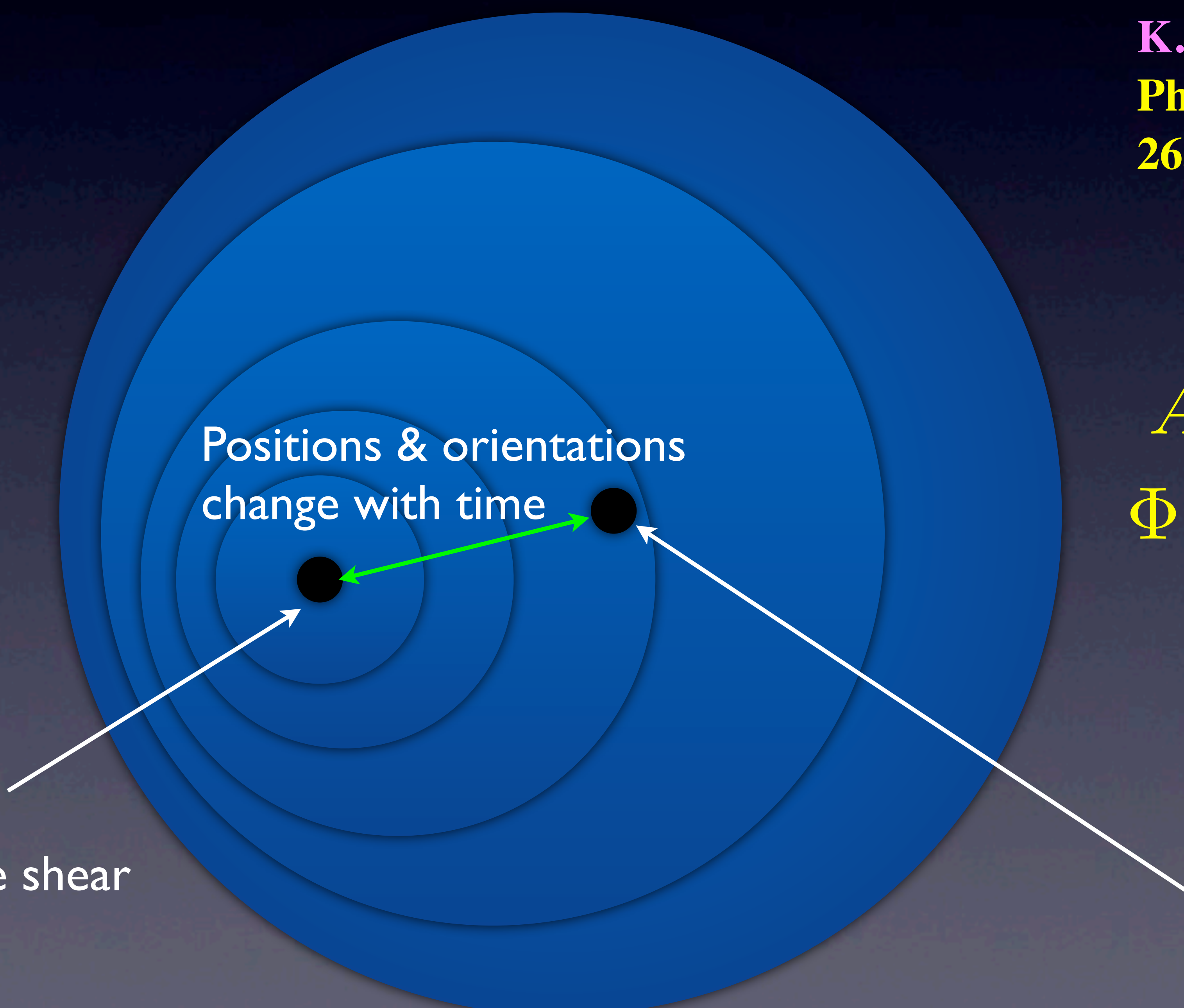


Figure 5. The density profile for diffrent time instants: a — 1 Gy after the Big Bang, b — 5.5 Gy, c — 10 Gy, d — present instant.

Without spherical symmetry: the "center" position is no longer unique --- NO need to do "fine-tuning"

In Szekeres quasi-spherical geometry ---- 2 possible "center" locations whose position & orientation changes with time:

K. Bolejko & R. A. Sussman
Physics Letters B 697 (2011) 265-270



Local isotropic observer where shear vanishes (r = 0)

Geometric center of 2-sphere of radius Phi = 250 Mpc

$$A = 4\pi\Phi^2(t, r)$$

$$\Phi(t, r_b) = 250 \text{ Mpc}$$

What needs to be done ??

Integrate null geodesics for the Szekeres Swiss cheese, and verify the fitting of SN Ia & CMB data

Test the KSZ effect with Szekeres

Difficult because there are no "radial null geodesics"

Much harder work !! (likely will not be done)

Current status in the use of inhomogeneous models to explain cosmic acceleration.

- Spherical Gpc voids are practically ruled out.
- Szekeres voids improve the fitting of observations but perhaps not much (must be tested).
- More general inhomogeneity requires 3d numerical codes & (likely) include small corrections from non-adiabatic and vector/tensor modes.
- There is a consensus in the community that inhomogeneity is no longer favored.

There are other ideas "floating" in the literature

- Include radiation. Effect: modifies initial conditions, may have effects on CMB fitting

Woei Chet Lim, Marco Regis, Chris Clarkson, JCAP 10 (2013) 010 [arXiv:1308.0902]

- Perturbations on an LTB background. Effect: allows for a more consistent probing of inhomogeneity

February et al, Class. Quantum Grav. 31 (2014) 175008

inhomogeneity in velocities instead of in densities?

- Consider effects of peculiar with respect to the Hubble flow. Effects: KSZ becomes more nuanced

David L Wiltshire et al, Hubble flow variance and the cosmic rest frame Phys. Rev. D 88, 083529 (2013)

C Tsagas, Peculiar motions, accelerated expansion and the cosmological axis, Phys. Rev. D 84, 063503 (2011)

other ideas ?

- Consider averages & coarse graining.

David L. Wiltshire What is dust? - Physical foundations of the averaging problem in cosmology Class.Quant.Grav.28:164006, 2011 arXiv:1106.1693

If inhomogeneous models cannot explain cosmic acceleration, then why do we need them?

We have become too fixed on the idea that considering inhomogeneous models IMPLIES refuting Dark Energy or Λ

However: the Universe can still be inhomogeneous with $\Lambda > 0$

NOTICE: the fact that $\Lambda > 0$ does NOT imply a Lambda-CDM Universe

Inhomogeneous models (with $\Lambda > 0$) can still be useful for tackling many problems

- Check if observations can be fit with an inhomogeneous model with $\Lambda > 0$ (a Λ-LTB model). Marra et al:

  ✓ Testing the Copernican principle by constraining spatial homogeneity, Wessel Valkenburg, Valerio Marra, Chris Clarkson, Mon.Not.Roy.Astron.Soc. 438 (2014) L6-L10 [arXiv:1209.4078v3 [astro-ph.CO]

  ✓ Observational constraints on the LLTB model, Valerio Marra, Mikko Paakkonen, JCAP12(2010)0 [arXiv1009.4193]

- Observational effect of inhomogeneities: they give the false "impression" of a varying Λ. Romano et al:

  ✓ Non perturbative effects of primordial curvature perturbations on the apparent value of a cosmological constant, Antonio Enea Romano, Sergio Sanes, Misao Sasaki, Alexei A. Starobinsky EPL, [arXiv:1311.1476]

- Theoretical issues: non-locality and averaging,

- Better understanding of alternative gravity theory

- Provide a theoretical framework for non-linear perturbations
- Examine relativistic & non-linear effects in structure formation and growth suppression

Exact solutions as "exact" perturbations: look at the following hierarchy

exact solution?

↑

non-linear perturbations

↑

Linear perturbations

Hidalgo & Sussman, work in progress

Szekeres $\{\rho,\ \mathcal{H},\ K,\ \Sigma,\ \mathcal{E}\}$

↓ ↓ ↓ ↓

FLRW $\{\rho,\ \mathcal{H},\ K\}$ $\{\Sigma = \mathcal{E} = 0\}$

Propose a solution based on assuming "EXACT" perturbation forms:

$$\rho = \rho_q \left[1 + \delta^{(\rho)}\right], \qquad \mathcal{H} = \mathcal{H}_q \left[1 + \delta^{(\mathcal{H})}\right]$$

where: $\{\,\rho_q\,,\,\mathcal{H}_q\,\}$ **are SZEKERES scalars that satisfy FLRW dynamics**

→ **"background" variables**

and: $\{\,\delta^{(\rho)}\,,\,\delta^{(H)}\,\}$ **are obtained from the 1+3 system**

→ **exact "perturbations"**

Look at the dynamics of these perturbations & compare with "standard" perturbations

The perturbations compare local covariant scalars with their weighed average that satisfies FLRW dynamics

$$A_q = \frac{\int_{\mathcal{D}} A\,\mathcal{F}\,d\mathcal{V}}{\int_{\mathcal{D}} \mathcal{F}\,d\mathcal{V}}$$

average with weight factor $\mathcal{F} = \left[\dot{R}^2 + \left(1 - \frac{2M}{R}\right)\right]^{1/2}$

$$d\mathcal{V} = \sqrt{\det(h_{ab})}\,d^3x$$

Proper volume

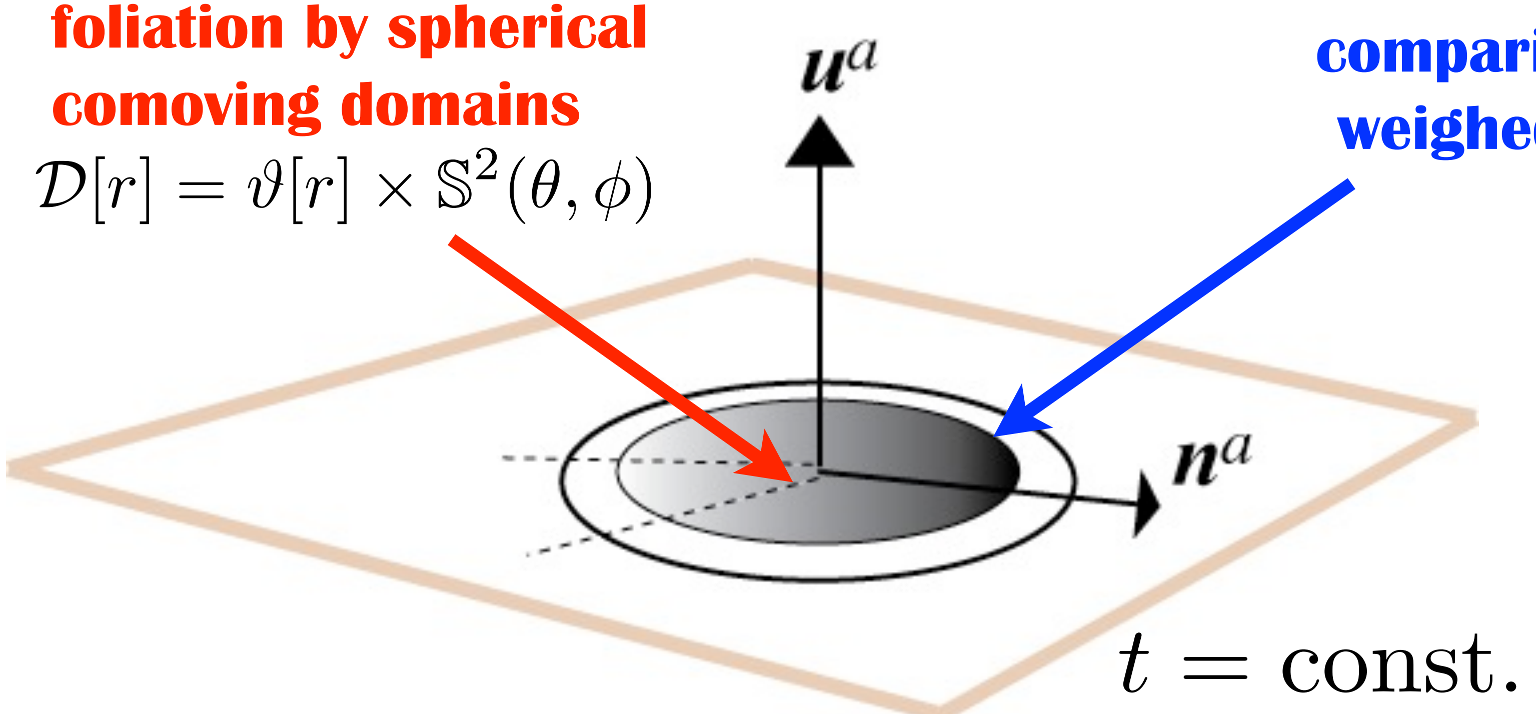
FLRW background defined in terms of averaged scalars

foliation by spherical comoving domains

$$\mathcal{D}[r] = \vartheta[r] \times \mathbb{S}^2(\theta, \phi)$$

comparison between local value A and weighed average A_q at each 2-sphere

$$\delta^{(A)} = \frac{A - A_q}{A_q}$$



We transform Szekeres dynamics into evolution equations for EXACT & COVARIANT perturbations on FLRW:

$$\dot{\rho}_q = -3\,\rho_q\,H_q,$$

$$\dot{H}_q = -H^2 - \frac{4\pi}{3}\rho_q,$$

background variables

$$\dot{\delta}^{(\rho)} = -3\,\left(1+\delta^{(\rho)}\right)\,H_q\,\delta^{(H)}$$

$$\dot{\delta}^{(H)} = -\left[\left(1+3\delta^{(H)}\right)\,\delta^{(H)} - \frac{\Omega_q}{2}\left(\delta^{(H)}-\delta^{(\rho)}\right)\right]H_q,$$

exact perturbations

$$H_q^2 = \frac{8\pi}{3}\rho_q - K_q, \qquad \Omega_q = \frac{8\pi\rho_q}{3H_q^2}$$

$$2\,\delta^{(H)} = \Omega_q\,\delta^{(\rho)} + (1-\Omega_q)\,\delta^{(K)}$$

$$\delta^{(\Omega)} = \delta^{(\rho)} - 2\delta^{(H)}$$

constraints

Reduce to standard perturbations in the linear limit

Algebraic constraints: → **Autonomous ODE's:**

DYNAMICAL SYSTEM !!

Growth suppression factor

Linear perturbations on FLRW

The growth suppression factor defined for linear perturbations (in an isochronous gauge) on a dust FLRW background with $\Lambda > 0$ or $\Lambda = 0$ is

$$f = \frac{d(\ln \delta)}{d(\ln \bar{a})} = \frac{(\ln \delta)^{\dot{}}}{(\ln \bar{a})^{\dot{}}} = \frac{\dot{\delta}/\delta}{\dot{\bar{a}}/\bar{a}}, \tag{1}$$

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}, \qquad \frac{\dot{\bar{a}}}{\bar{a}} = \frac{\bar{\Theta}}{3} = \bar{H}, \tag{2}$$

where $\bar{a}$, $\bar{\rho}$, $\bar{H}$ are the scale factor, density and Hubble scalar of the FLRW background and δ is the density contrast satisfying the linear equation

$$\ddot{\delta} + 2\bar{H}\dot{\delta} - 4\pi\bar{\rho}\delta = 0. \tag{3}$$

Exact dust perturbations (LTB & Szekeres)

$$\ddot{\delta}_q^{(\rho)} - \frac{2\left[\dot{\delta}_q^{(\rho)}\right]^2}{1+\delta_q^{(\rho)}} + 2H_q\dot{\delta}_q^{(\rho)} - 4\pi\rho_q\delta_q^{(\rho)}(1+\delta_q^{(\rho)}) = 0,$$

Relation between exact perturbations vs curvature & kinematic invariants

$$\sigma_{ab} = \Sigma\,\mathbf{e}_{ab}, \qquad \Sigma = -(H - H_q) = -H_q\delta_q^{(H)},$$

$$E_{ab} = \Psi_2\,\mathbf{e}_{ab}, \qquad \Psi_2 = \frac{4\pi}{3}(\rho - \rho_q) = \frac{4\pi}{3}\rho_q\delta_q^{(\rho)},$$

$$\delta_q^{(\rho)} = -\frac{\phi}{1+\phi}, \qquad \phi = \frac{6\Psi_2}{\mathcal{R}},$$

$$\delta_q^{(H)} = \frac{\xi}{1-\xi}, \qquad \xi = \frac{\Sigma}{H},$$

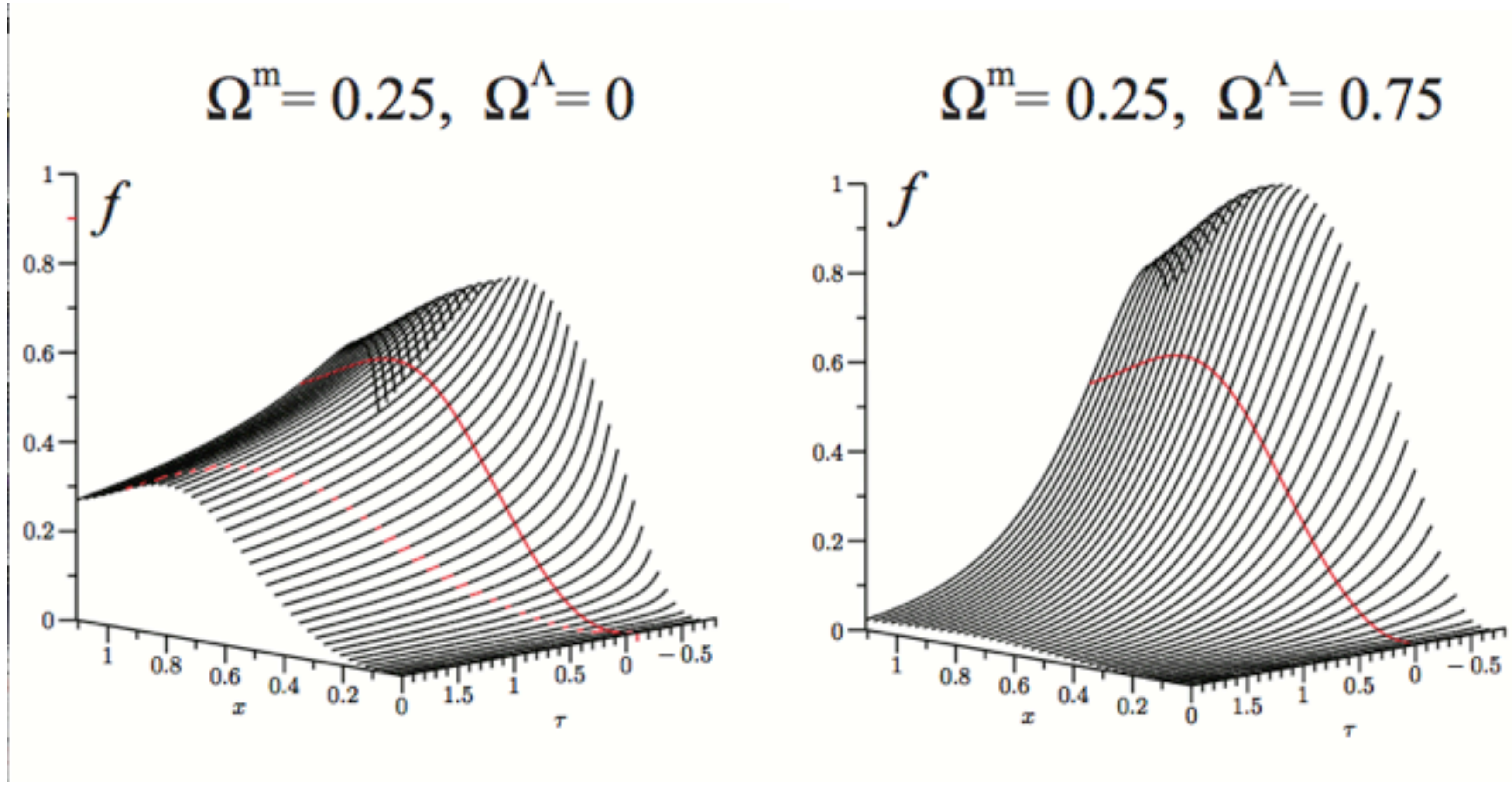
$\mathcal{R}$ **Ricci Scalar** $\quad \Psi_2$ **Weyl conformal invariant**

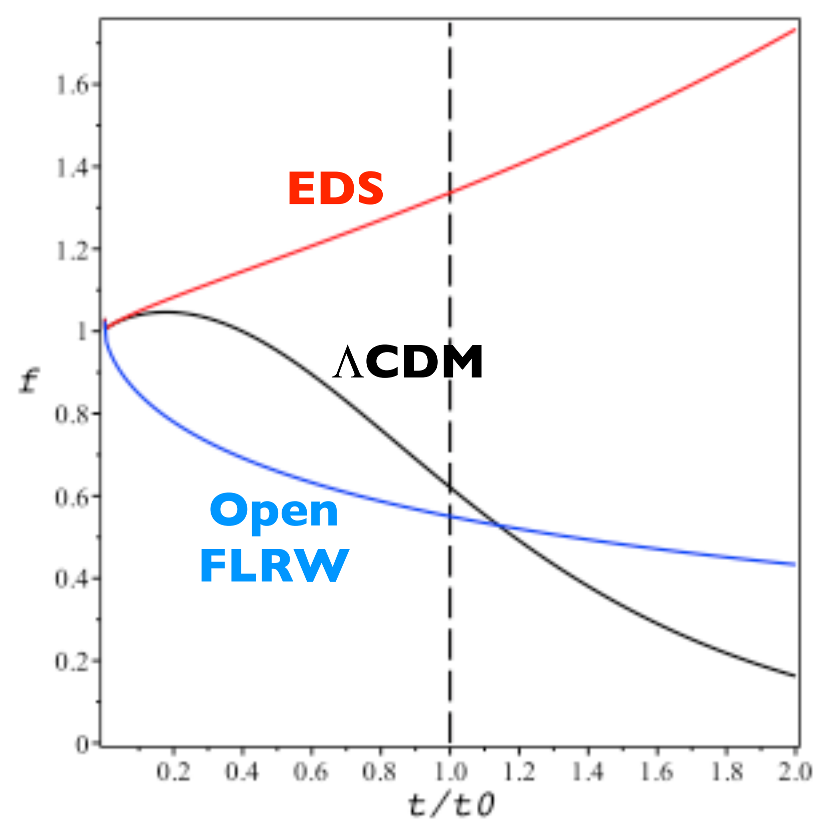
Invariant meaning of growth suppression factor

$$f = -\frac{3\,\xi}{\phi} = -\frac{\Sigma/H}{2\Psi_2/\mathcal{R}},$$

Ratio: anisotropy of expansion vs Weyl/Ricci curvature

Numerical results for 50 Mpc LTB voids





Λ introduces a strong suppression effect, but may not be noticeable in our cosmic time t = t0 (more discussion needed)

THANKS FOR YOUR ATTENTION